Mathematical Modeling and Simulation of Permanent Magnet Synchronous Machine

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***Abstract*—Rapid developments are occurring in design of electrical machines and its control. To get optimum performance under normal/fault condition needs implementation of fault tolerance. Next decade is of Permanent Magnet Synchronous Machines. The paper presents mathematical modeling of PMSM. A current and speed controller is designed to implement fault tolerance and its stability analysis.**

***Index Terms*—PMSM, fault tolerance, mathematical model**

I. Introduction

Performance of motor control systems contribute to a great extent to the desirable performance of drive. Existing motors in the market are the Direct Current and Alternative Current motors, synchronous and asynchronous motors. These motors, when properly controlled and excited produce constant instantaneous torque

A. *Objective of Problem*

In what respect PMSM are better and whether motor be designed from few watts to mega watt with stable operation

B. *Classification of Faults in PMSM*

Replacing rotor field windings and pole structure with permanent magnets makes the motor as brushless motors. Permanent Magnet Synchronous Machines (PMSM) can be designed with any even number poles. Greater the number of poles produces greater torque for the same level of current [1]. Surface Mounted PMSM and Interior PMSM are two types of PMSMs. In SPMSM magnets are mounted on the rotor, air gap is large hence armature reaction is negligible. Whereas in IPMSM, magnets are buried inside the rotor,rotor is robust and airgap is uniform hence suitable for high speed operation. Most frequently occurring faults in PMSM can be classified as:

- Faults related to electrical structure
- Faults related to mechanical structure

The survey indicates that the stator related faults are major faults around 35-40% and are related to the stator winding insulation and core. The terminal connector failure may also result in winding faults. Stator winding related failures can be divided into winding open circuit fault, winding short circuit fault and winding turn to turn short circuit fault, winding to ground short circuit fault. These faults are the consequence of long term or overstressed operation in the motor drive. The second set of faults is magnetic or mechanical. Among these faults are partial demagnetization of the rotor magnets, rotor eccentricity.

Feature of a fault tolerant system is the ability to detect the fault and then to clear it. Based on this, the optimal excitation for the stator winding has to be calculated. This data is stored in the memory and in case of a fault the optimal currents would be applied to the remaining healthy phases to get the maximum torque per ampere in the output. This needs deriving a mathematical model in suitable form [2].

II. Mathematical Model of PMSM

In PMSM the inductances vary as a function of the rotor angle. The two-phase (d-q) equivalent circuit model is used for analysis. The space vector form of the stator voltage equation in the stationary reference frame is given as:

$$v_s = r_s i_s + d\lambda_s/dt \tag{2.1}$$

where r_s , V_s , i_s , and λ_s are the resistance of the stator winding, complex space vectors of the three phase stator voltages, currents, and flux linkages, all expressed in the stationary reference frame fixed to the stator, respectively. They are defined as:

$$\begin{aligned} v_s &= [v_{sa}(t) + a v_{sb}(t) + a_2 v_{sc}(t)] \\ i_s &= [i_{sa}(t) + a i_{sb}(t) + a_2 i_{sc}(t)] \\ \lambda_s &= [\lambda_{sa}(t) + a\lambda_{sb}(t) + a_2\lambda_{sc}(t)] \end{aligned} \tag{2.2}$$

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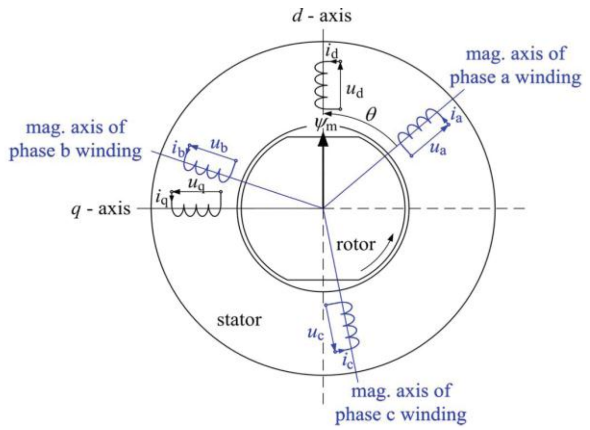

Figure 1. Schematic representation of a two-pole, three-phase permanent magnet synchronous machine.

Fig. 1 illustrates a conceptual cross-sectional view of a three-phase, two-pole surface mounted PMSM along with the two-phase d-q rotating reference frame. Symbols used in (2.2) are explained in detail below:*a*, and a_2 are spatial operators for orientation of the stator windings, $a = e^{j2\pi/3}$ and $a_2 = e^{j4\pi/3}$. v_{sa}, v_{sb}, v_{sc}. are the values of stator instantaneous phase voltages. i_{sa}, i_{sb}, i_{sc} are the values of stator instantaneous phase currents. λ_{sa}, λ_{sb}, λ_{sc} are the stator flux linkages and are given by:

$$\begin{aligned}\lambda_{sa} &= L_{aa}i_a + L_{ab}i_b + L_{ac}i_c + \lambda_{ra}\\ \lambda_{sb} &= L_{ab}i_a + L_{bb}i_b + L_{bc}i_c + \lambda_{rb}\\ \lambda_{sc} &= L_{ac}i_a + L_{bc}i_b + L_{cc}i_c + \lambda_{rc}\end{aligned} \tag{2.3}$$

where L_{aa}, L_{bb}, and L_{cc}, and are the self-inductances of the stator a,b and c-phase respectively. L_{ab}, L_{bc} and L_{ca}, are the mutual inductances between the ab, bc and ac phases respectively [3]. λ_{ra}, λ_{rb} and λ_{rc} are the flux linkages that change depending on the rotor angle established in the stator a, b, and c phase windings respectively due to the presence of the permanent magnets on the rotor. They are expressed as:

$$\begin{aligned}\lambda_{ra} &= \lambda_r \cos\theta\\ \lambda_{rb} &= \lambda_r \cos(\theta - 120°)\\ \lambda_{rc} &= \lambda_r \cos(\theta + 120°)\end{aligned} \tag{2.4}$$

In equation (2.4), λ_r represents the peak flux linkage due to the permanent magnet. It is often referred to as the back-emf constant. Note that in the flux linkage equations, inductances are the functions of the rotor angle. Self-inductance of the stator a-phase winding, L_{aa} including leakage inductance and a and b-phase mutual inductance,

$L_{ab} = L_{bc}$ have the form:

$$\begin{aligned}L_{aa} &= L_{ls} + L_0 - L_{ms}\cos(2\theta)\\ L_{ab} &= L_{ba} = ½ L_0 - L_{ms}\cos(2\theta)\end{aligned} \tag{2.5}$$

where L_{ls} is the leakage inductance of the stator winding due to the armature leakage flux.L_0 is the average inductance due to the space fundamental air-gap flux; $L_0 = ½ (L_q + L_d)$, L_{ms} is the inductance fluctuation (saliency); due to the rotor position dependent on flux $L_{ms} = ½ (L_d - L_q)$.Similar to that of L_{aa} but with θ replaced by $(\theta\text{-}2\pi/3)$ and $(\theta\text{-}4\pi/3)$for b-phase and c-phases self-inductances. L_{bb} and L_{cc}, can also be obtained similarly. All stator inductances are represented in matrix form below:

$L_{ss}=$

$L_{ls} + L_0 - L_{ms}\cos 2\theta \quad -.5L_0 - L_{ms}\cos 2(\theta-\pi/3) \quad .5L_0 - L_{ms}\cos 2(\theta+\pi/3)$

$-.5L_0 - L_{ms}\cos 2(\theta-\pi/3) \quad L_{ls} + L_0 - L_{ms}\cos 2(\theta-2\pi/3) \quad -.5L_0 - L_{ms}\cos 2(\theta-\pi)$

$-.5L_0 - L_{ms}\cos 2(\theta+\pi/3) \quad -.5L_0 - L_{ms}\cos 2(\theta+\pi) \quad L_{ls} + L_0 - L_{ms}\cos 2(\theta+2\pi/3)$

(2.6)

It is evident from the above equation that the elements of L_{ss} are a function of the rotor angle which varies with time at the rate of the speed of rotation of the rotor. Under a three-phase balanced system with no rotor damping circuit and knowing the flux linkages, stator currents, and resistances of the motor, the electrical three-phase dynamic equation in terms of phase variables can be arranged in matrix form similar to that of (2.1) and written as:

$$[v_s]=[r_s][i_s]+d/dt[\lambda_s] \tag{2.7}$$

where, $[v_s] = [v_{sa}, v_{sb}, v_{sc}]t$

$$\begin{aligned}[I_s] &= [i_{sa}, I_{sb}, I_{sc}]t\\ [r_s] &= [r_a, r_b, r_c]t\\ [\lambda_s] &= [\lambda_{sa}, \lambda_{sb}, \lambda_{sc}]t\end{aligned} \tag{2.8}$$

In (2.7), v_s, i_s, and λ_s refer to the three-phase stator voltages, currents, and flux linkages in matrix forms as shown in equation(2.8) respectively. Further all the phase resistances are equal and constant, $r_s = r_a = r_b = r_c$

A. Park's Transformation

As L_{ss} presents computational difficulty (in equation, 2.1) when used to solve the phase quantities directly. To obtain the phase currents from the flux linkages, the inverse of the time varying inductance matrix will have to be computed at every time step [4]. The computation of the inverse at every time step is time-consuming and could produce numerical stability problems. To remove the time-varying quantities in voltages, currents, flux linkages and phase inductances, stator quantities are transformed to a d-q rotating reference frame using Park's transformation. This results in the above equations having time-invariant coefficients. The idealized machines have the rotor windings along the d- and q-axes. Stator winding quantities need transformation from three-phase to two-phase d-q rotor rotating reference frame. Park's transformation is used to transform the stator quantities to d-q reference frame, the d-axis aligned with the magnetic axis of the rotor and q-axis is leading the d axis by $\pi/2$ as shown in Fig.1. The original d-q Park's Transformation $[T_{dq0}\theta r]$ is applied to the stator quantities n in equation 2.7 is given by:

$$\begin{aligned}[v_{dq0}] &= [T_{dq0}(\theta r)][v_s]\\ [i_{dq0}] &= [T_{dq0}(\theta r)][i_s]\\ [\lambda_{dq0}] &= [T_{dq0}(\theta r)][\lambda_s]\end{aligned} \tag{2.9}$$

Where

$$[v_{dq0}]=[v_d v_q v_0] \tag{2.10}$$

$$[v_{dq0}]=[T_{dq0}(\theta_r)][r_s][T_{dq0}(\theta_r)]^{-1}[i_{dq0}]+[T_{dq0r})]p[T_{dq0}(\theta_r)]^{-1}[_{dq0}] \tag{2.11}$$

$$[T_{dq0}\ 6\ (\theta_r)][r_s][T_{dq0}(\theta_r)]^{-1}[i_{dq0}]=r_s[i_{dq0}] \tag{2.12}$$

The model of PMSM without damper winding has been developed using the following assumptions:

- Saturation is neglected.
- The induced EMF is sinusoidal.
- Eddy currents and hysteresis losses are negligible.
- There are no field current dynamics.

Voltage equations are given by:

$$v_d = R_s\, i_d - \omega_r \lambda_q + \frac{d\lambda d}{dt}$$
$$v_q = R_s i_q - \omega_r \lambda_d + \frac{d\lambda q}{dt} \qquad (2.18)$$

$$\lambda_{d\lambda} = L_d i_d + \lambda_f$$
$$\lambda_q = L_q i_q \qquad (2.19)$$

Substituting equations 2.19 in 2.18:

$$V_d = R_s i_d - \omega_r L_q i_q + (L_d i_d + \lambda_f)$$
$$V_q = R_s i_q - \omega_r L_q i_q + (L_q i_q) \qquad (2.20)$$

The developed torque is given by

$$T_e = 1.5(P/2)(\lambda_d i_q - \lambda_q i_d) \qquad (2.22)$$

The mechanical Torque equation is

$$T_m = T_L + B\omega_m + J(d\omega_m/dt) \qquad (2.23)$$

Solving for the rotor mechanical speed from equation 2.23

$$\omega_{m=} \ ([(T_e - T_L - B\omega_m)/J]dt$$
$$\omega_m = \omega_r(2/P) \qquad (2.24)$$

In the above equations ω_r is the rotor electrical speed where as ω_m is the rotor mechanical speed.

III. Mathematical Model of Reference Current Calculation

The back emf voltages of an *n*-phase fault tolerant PM motor drive can be given as

$$e_j = K_e\, \omega\, e_j\, (\theta) \qquad (3.1)$$

Here j is an integer representing the phase number of the motor(j= 1,2,3…*n*.) The output torque T_0 with two phases can be expressed as a function of the phase currents and corresponding back emf voltages as

$$T_0 = K_e[e_1 i_1 + e_2 i_2] \qquad (3.2)$$

Here i_1 and i_2 are instantaneous phase currents. From equation (1.2) current i_2 can be derived as:

$$i_2 = [(T_0/k_e) - e_1 i_1]/e_2$$
$$P_{cu} = (i_1^2 + i_2^2)R \qquad (3.3)$$

R is resistance of phase winding. For copper loss to be minimum, $Y=(i_1^2+i_2^2)$ should be minimum

$$Y = i_1^2 + (1/e_2^2)[(T_0^2/k_e^2) - 2*(T_0/k_e)e_1 i_1 + e_1^2]$$
$$i_1 = [e_1/(e_1^2+e_2^2)]\,(T_0/k_e) \qquad (3.4)$$
$$i_2 = [e_2/(e_1^2+e_2^2)]\,(T_0/k_e)$$

A. Fault Analysis in Fault Tolerant Motor Drives

Winding open circuit fault is the most common fault and detected by the phase current sensor when the phase current is zero. Desirable currents for zero torque ripple operation assuming $i_1 = Im\ sin\ \theta$, $i_2 = Im\ sin(\theta - 120^0)$, $i_{3=} Im\ sin(\theta + 120^0)$

Here, Im is peak value of phase current [5]. Hence for healthy operation the total output torque for single motor drive is $T_0 = (3/2) K_e I_{m0.}$

If phase 1 is open circuited and no fault remedial strategies are adopted, the total output torque is given by $T_{f0} = K_e Im(1+0.5\cos 2\theta)$

Here T_{f0} is torque under faulty condition

B. Winding Short Circuit Fault Analysis

The winding short circuit fault is the most critical fault. Equivalent circuit of winding short circuit fault and switch short circuit fault and its waveforms are shown in Fig. 2 and Fig. 3.

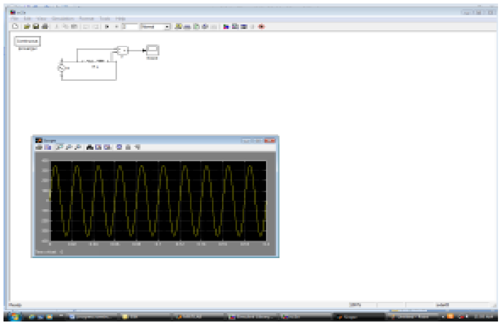

Figure 2. Winding short circuit fault waveform.

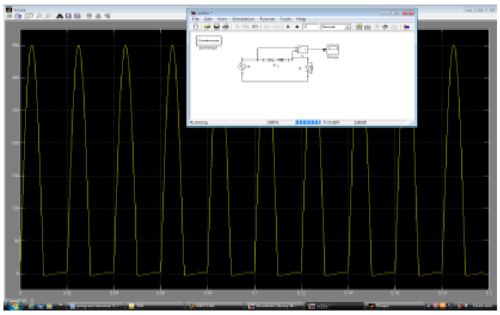

Figure 3. Switch short circuit fault waveform.

For a sinusoidal back emf, during short circuit fault operation, a short circuit current I_{sc} will be generated in the winding and the peak steady state current is given by

$$I_{scm} = E_m/(\surd(R^2 + [\omega_m L]^2$$

But E_m is function of rotor speed and back emf constant $I_{sc} = K_e\omega_m / (\surd(R^2+[\omega_m L]^2$

Corresponding copper loss is expressed as

$$P_{cusc} = (I_{scm2}/\surd 2)^2$$
$$T_{drag} = I_{sc}\, E_m \cos\varphi\, /2\omega_m,\ \cos\varphi = R/(\surd(R^2+[\omega_m L]^2$$
$$T_{drag} = I_{sc}\, E_m \cos\varphi\, /2\omega_m = K_e^2\, R\, \omega_m /2 * ((R^2+[\omega_m L]^2)$$

Assuming base voltage is equal to back emf voltage at rated rotor speed. Base current is equal to winding short circuit current

$$I_{sc} = \omega_m / (\surd R_{pu}^2 + (\omega_{pu} + L_{pu})^2$$
$$T_{avedrag} = \omega_{pu} R_{pu} / R_{pu}^2 + (\omega_{pu} + L_{pu})^2$$
$$P_{cuscu} = \omega_{pu}^2 R_{pu} / R_{pu}^2 + (\omega_{pu} + L_{pu})^2$$

Where R_{pu} and L_{pu} represent the per unit resistance and the inductance of the short circuited windings. ω_{pu} is the per unit value of the angular speed the motor. This needs reference current controller to be designed [6].

C. Current Controller Design

The error between reference speed and measured speed is send to a PI controller. Since d and q axis controller are identical tuning of q axis controller is sufficient. Structure of q axis current controller is shown in Fig. 4.

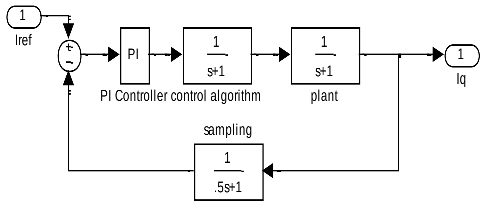

Figure 4. Q axis current controller.

PI controller offers a zero steady state error. Its transfer function is $Gc(s)=U(s)/E(s)$

$$= k_{pi}+k_{ii}/s$$

$$= k_{pi}\,(1+T_{ii}s)/\,T_{ii}s$$

where k_{pi} is proportional gain, K_{ii} is integrator gain T_{ii} integrator time constant. It is a ratio between K_{pi} and K_{ii}

$$T_{ii}= K_{pi}/\,K_{ii}\,,T_s=0.2\ ms$$

$$G_{pe}(s)= i_q(s)/U_q(s)$$

$$=1/R_s+sL_s=1/\,R_s\,(1+s\,L_s/R_s)$$

$$= K/(1+sT_q)\ k=1/R_s\ R_s\text{-stator resistance}$$

$T_q = L_s/R_s$ =0.116sec is the time constant of the motor

The feedback is the delay introduced by digital to analog conversion. It is a first order transfer function with time constant T_s

$$G_{sam}(s)= 1/(0.5\ T_s\ s+1)$$

Thus modified q axis controller with unity feedback is shown in Fig. 5 below [7].

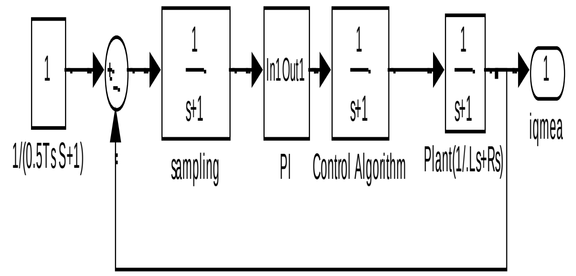

Figure 5. Modified q axis current controller.

The open loop transfer function is given by

$$G_{OL}(s)=G_{sam}(s)*Gc(s)*G_{CA}(s)G_{PL}(s)$$

$$=[1/(0.5T_ss+1)]*[k_{pi}(1+T_{ii}s)/T_{ii}s]\,[1/(T_ss+1)]*[\ K/(\ 1+sT_q)]$$

The slowest pole is the one of PMSM transfer function and is meant to cancel the zero of controller, making the system more stable. This implies

$$(1+T_{ii}s)\rightarrow (1+sT_q)\rightarrow T_{ii}$$

$$T_q = L_s/R_s = 0.116sec$$

In order to simplify the transfer function a time constant is introduced [8]. It represents the approximation of first order transfer function that introduce delays. Its transfer function will be replaced by a unique transfer function of first order having the time constant equal to the sum of all the time constants from the system.

$$T_{si} = 1.5T_s=0.3msec$$

With above assumptions the open loop transfer function of second order system has the form

$$G_{om}(s)=1/2\zeta s(\zeta s+1)$$

$$K_{pi}\,K/T_{ii} =(1/2*T_{si})$$
$$= K_{pi}$$
$$= (T_{ii}\,R_s\,)/(2*T_{si})$$
$$=3.333$$

Thus Pi transfer function is

$$Gc(s)=3.33+(300/s)$$

The bode diagram of the q axis current controller is shown in Fig. 6.

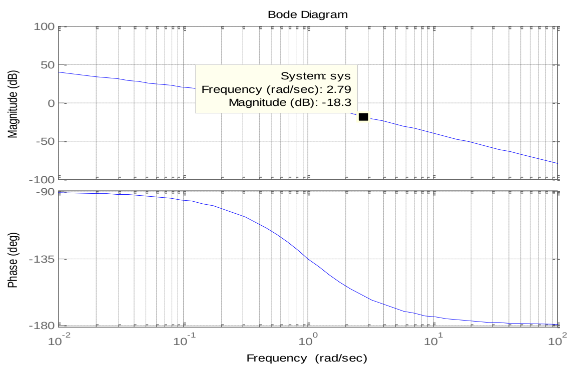

Figure 6. Bode plot of q axis controller .

GM=19.1dB, PM=63.6,
Closed loop system stable? yes
Maximum overshoot Mp= 4.32 %
Settling time t_s=2.53 ms , Rise time t_r= 0.912 ms

D. Design of Speed Controller

The speed controller block diagram is shown below
The transfer function of Pi controller is given by

$$Gc(s)= K_{ps}(1+T_{is}s)/\,T_{is}s$$

Where K_{ps} is the proportional gain of the speed controller and T_{is} is the time constant. The delay introduced by the digital calculations is representing the control algorithm

[9]. Transfer function has the form of a first order system with time constant Ts=0.2ms=1/5k=1/fs, $G_{A(s)}=1/1+T_s s$

The current control loop can be expressed like a first order system $T_{iq}=T_{ii}\, R_s/K_{pi}$

$$T_F = Gc(s)=1/(1+T_{iq}s)$$

The mechanical equation of PMSM is given by

$$T_m=Jdw_m/dt+Bw_m+T_L$$

T_L is the load torque and is considered as disturbance hence will not be considered. Filter has time constant $T_f=1/w_c=1/2\pi f=0.796$ *ms*

The sampling block introduces delay for digital to analog conversion and is equal to Ts,

$$G_{sam}(s)=1/0.5T_s s+1$$

Thus sampling the T_F, the disturbances are not taken into consideration. The closed loop system is modified with unity feedback as shown below [10].

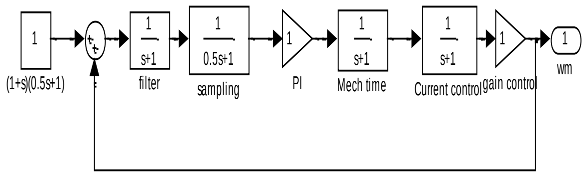

Figure 7. Modified closed loop current control system.

Its open loop transfer function:

$$G_{OL}(s)=[1/(0.5T_s s+1)(T_{fs}s+1)][K_{ps}(1+T_{is}s)/_{is}s)]*[1/T_s s+1][\,1/(1+T_{iq}s\,)]$$

In order to simplify the transfer function all the time constants of the delays are approximated with one time constant:

$$T_{ss}=1.5T_s+T_f+T_{iq} = 1.8 \text{ ms}$$

So, the open loop transfer function becomes:

$$G_{OL}(s)=k_c\, K_{ps}\,(1+T_{is}s)/JT_{is}s^2(1+T_{ss}s)$$

With $k_c=1.5*\lambda_m$, $K_{ps}=1/k_1T_1=0.462$,

$T_{is}=4T_1=4T_{ss}$

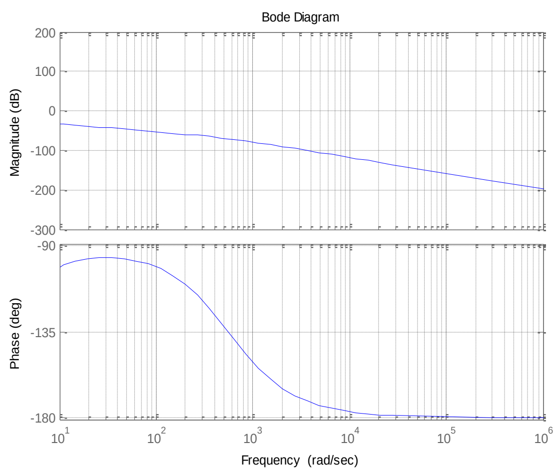

Figure 8. Bode plot for modified system.

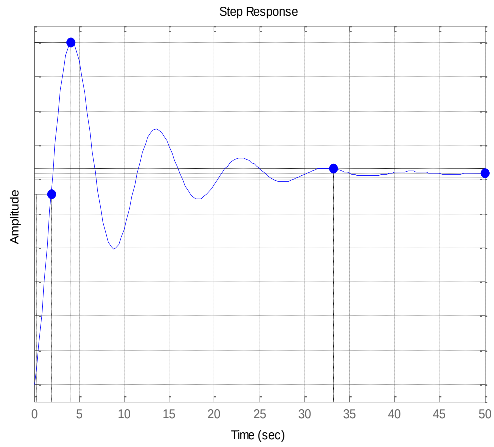

Figure 9. Step response for modified system.

Its bode plot and step response is shown in Fig. 8 and Fig. 9.

GM=13(dB) at 689 rad/sec
PM=34.89deg) at 300 rad/ sec
Closed loop system is stable? yes

IV. Conclusion

The mathematical model helps to design reference current controller needed to design fault tolerant system. The design of the current and speed controllers are necessary for good implementation of control strategy. As shown by bode plot for current and speed controller, the system is second order hence stable. Further use of permanent magnets helps in reduction of size and gives higher current, torque and speed compared to machine with electromagnets. The only drawback is higher cost of magnets.

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