A Nonlinear Observer via Augmented Linear System based on Formal Linearization using Discrete Fourier Expansion

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***Abstract*—In order to expand a formal linearization method on designing an observer for nonlinear dynamic and measurement systems, we exploit the discrete Fourier expansion that can reduce computational complexity. A nonlinear multidimensional dynamic system is considered by a nonlinear ordinary differential equation, and a measurement equation is done by a nonlinear equation. Defining a linearization function which consists of the trigonometric functions considered up to the higher-order, the nonlinear dynamic system is transformed into an augmented linear one with respect to this linearization function by using the discrete Fourier expansion. Introducing an augmented measurement vector which consists of polynomials of measurement data, the measurement equation is transformed into an augmented linear one with respect to the linearization function in the same way. To these augmented linearized systems, a linear estimation theory is applied to design a nonlinear observer.**

***Index Terms*—nonlinear system, nonlinear observer, formal linearization, discrete Fourier expansion**

I. INTRODUCTION

There exits an estimation problem of the state of nonlinear systems on the basis of output data which may not provide enough information to determine the state of the systems. Measurement equations might be given by sticky nonlinear functions. The design of observers for such nonlinear systems is not easily made and is less understood than those for linear systems. Linearization is one of approaches in order to apply the linear system theories [1]-[6]. Formal linearization [7]-[12] is one of them to treat with these nonlinear problems.

In the previous work, a nonlinear observer design using the formal linearization method based on the Fourier expansion was considered [11]. In this paper, we develop a nonlinear observer via the formal linearization method based on the discrete Fourier expansion in order to reduce computational complexity. Introducing a linearization function which consists of the trigonometric functions of the state variables, and then composing an augmented measurement vector which consists of polynomials of the measurement variables, a given nonlinear system is transformed into an augmented linear system by using the discrete Fourier expansion. To this formal linear system, a linear estimation theory is applied to derive a nonlinear observer. In this approach, its inversion is obtained by simple calculation of the combination of the trigonometric functions.

One of advantages in this method is that coefficients of linearized system are simply obtained by carrying out summation due to the orthogonality for a finite sum. Because the previous work [11] must be executed by the complicated integral calculus to get them.

Numerical experiments are illustrated to verify the effectiveness of this observer design in comparison with a conventional linearization based on Taylor expansion truncated at the first order.

II. PRELIMINARIES

In this section, we explain an estimation problem. Suppose that a nonlinear dynamic system is described by a state differential equation

$$\Sigma_1 : \dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t)), \tag{1}$$

$$\boldsymbol{x}(0) = \boldsymbol{x}_0 \in \mathcal{D}$$

where t denotes time, $\dot{} = d/dt$, $\boldsymbol{x}$ is an $n\times 1$ state vector, and $\boldsymbol{f}$ is a sufficiently smooth nonlinear function. $\mathcal{D}$ is a compact domain denoted by the Cartesian product:

$$\mathcal{D} = \prod_{i=1}^{n} [l_i - p_i, l_i + p_i] \subset R^n$$

where l_i ($l_i \in R$) is the middle of the domain of x_i and p_i ($p_i > 0$) is half of the domain of x_i ($i = 1, \cdots, n$).

Let a measurement equation be described as

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$$\boldsymbol{\eta}(t) = \boldsymbol{h}(\boldsymbol{x}(t)) \in R^{\ell} \tag{2}$$

where $\boldsymbol{\eta}$ is an $\ell \times 1$ output vector with $\ell < n$, and $\boldsymbol{h}(\boldsymbol{x})$ is a sufficiently smooth nonlinear function.

The problem is that the state of the nonlinear dynamic system (1) can be estimated by use of the given nonlinear measurement output (2).

III. Nonlinear Observer by Formal Linearization

A. Formal Linearization for Dynamic System

In this paper, we exploit the discrete Fourier expansion by using a formal linearization method [11]. Since the basic domain of the Fourier expansion is

$$\mathcal{D}_0 = \prod_{i=1}^{n} [0, 2\pi] \tag{3}$$

and state vector $\boldsymbol{x}$ is changed into $\boldsymbol{y}$ by

$$\boldsymbol{y} = \pi P^{-1}(\boldsymbol{x} - L) + \pi I \in \mathcal{D}_0 \tag{4}$$

where

$$L = \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix}, P = \begin{pmatrix} p_1 & & 0 \\ & \ddots & \\ 0 & & p_n \end{pmatrix},$$

$$I = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}, \boldsymbol{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}.$$

The given dynamic system (1) becomes

$$\dot{\boldsymbol{y}}(t) = \pi P^{-1} \boldsymbol{f}\Big(P(\frac{1}{\pi}\boldsymbol{y}(t) - I) + L\Big). \tag{5}$$

The discrete Fourier expansion has the trigonometric functions $\{T_r(\cdot)\}$ defined by

$$T_0(y_i) = 1,$$

$$T_r(y_i) = \begin{cases} \cos\dfrac{r+1}{2}y_i \ , & r : \text{odd number} \\ \sin\dfrac{r}{2}y_i \ , & r : \text{even number} \end{cases}, \tag{6}$$

and their derivative of the trigonometric functions

$$S_r(y_i) \equiv \frac{dT_r(y_i)}{dy_i} \tag{7}$$

is given by

$$S_0(y_i) = 0,$$

$$S_r(y_i) = \begin{cases} -\dfrac{r+1}{2}\sin\dfrac{r+1}{2}y_i \ , & r : \text{odd number} \\ \dfrac{r}{2}\cos\dfrac{r}{2}y_i \ , & r : \text{even number} \end{cases}. \tag{8}$$

Using these trigonometric functions, an N-th order linearization function $\boldsymbol{\phi}(\cdot) = \boldsymbol{\phi}(\boldsymbol{y}(\cdot))$ which consists of the trigonometric functions is defined as

$$\begin{aligned} \boldsymbol{\phi} &= [\phi_1, \phi_2, \cdots, \phi_i, \cdots, \phi_{(2N+1)^n-1}]^T \\ &= [T_{(10\cdots0)}(\boldsymbol{y}), T_{(01\cdots0)}(\boldsymbol{y}), \cdots, T_{(0\cdots01)}(\boldsymbol{y}), \\ &\quad T_{(11\cdots0)}(\boldsymbol{y}), T_{(101\cdots0)}(\boldsymbol{y}), \cdots, T_{(10\cdots1)}(\boldsymbol{y}), \\ &\quad T_{(20\cdots0)}(\boldsymbol{y}), T_{(02\cdots0)}(\boldsymbol{y}), \cdots, T_{(r_1\cdots r_n)}(\boldsymbol{y}), \\ &\quad \cdots, T_{(2N\cdots2N)}(\boldsymbol{y})]^T \end{aligned} \tag{9}$$

where

$$T_{(r_1\cdots r_n)}(\boldsymbol{y}) = \prod_{i=1}^{n} T_{r_i}(y_i) \ .$$

The derivative of each element of ϕ along with the solution of the given nonlinear system (1) becomes

$$\begin{aligned} \dot{\phi}_\alpha(\boldsymbol{y}) &= \dot{T}_{(r_1\cdots r_n)}(\boldsymbol{y}) = \frac{\partial T_{(r_1\cdots r_n)}(\boldsymbol{y})}{\partial \boldsymbol{y}^T}\dot{\boldsymbol{y}} \\ &= [S_{r_1}(y_1)T_{r_2}(y_2)\cdots T_{r_{n-1}}(y_{n-1})T_{r_n}(y_n), \\ &\quad T_{r_1}(y_1)S_{r_2}(y_2)\cdots T_{r_{n-1}}(y_{n-1})T_{r_n}(y_n), \cdots, \\ &\quad T_{r_1}(y_1)T_{r_2}(y_2)\cdots T_{r_{n-1}}(y_{n-1})S_{r_n}(y_n)] \\ &\quad \times \pi P^{-1} \boldsymbol{f}\Big(P(\frac{1}{\pi}\boldsymbol{y}(t) - I) + L\Big) \\ &\equiv G_{(r_1\cdots r_n)}(\boldsymbol{y}), \ \alpha = \alpha(r_1, \cdots, r_n). \end{aligned} \tag{10}$$

Applying the discrete Fourier expansion which is considered up to the N-th order, this $G_{(r_1 \ldots r_n)}(\boldsymbol{y})$ is approximated by the trigonometric functions as

$$\hat{G}_{(r_1\cdots r_n)}(\boldsymbol{y}) \approx \sum_{q_1=0}^{2N} \cdots \sum_{q_n=0}^{2N} C_{(q_1\cdots q_n)}^{(r_1\cdots r_n)} T_{(q_1\cdots q_n)}(\boldsymbol{y}) \tag{11}$$

where the coefficients of the discrete Fourier expansion $C_{(q_1\cdots q_n)}^{(r_1\cdots r_n)}$ is computed using the following formula

$$\begin{aligned} C_{(q_1\cdots q_n)}^{(r_1\cdots r_n)} &\equiv \frac{2^{n-\gamma}}{\prod_{i=1}^{n}(2N+1)} \sum_{j_1=0}^{N}\sum_{j_2=0}^{N}\cdots \\ &\quad \sum_{j_n=0}^{N} G_{(r_1\cdots r_n)}(y_{1j_1}, y_{2j_2}, \cdots, y_{nj_n}) \\ &\quad \times T_{q_1}(y_{1j_1})T_{q_2}(y_{2j_2})\cdots T_{q_n}(y_{nj_n}) \ , \end{aligned} \tag{12}$$

$$\gamma = \{\text{the number of } q_i = 0 : 1 \le i \le n\},$$

and the sample points $\{y_{ij_i}\}$ are set to be

$$y_{ij_i} = \frac{2\pi}{2N+1}j_i, \ (i = 1, \cdots, n, \ j_i = 0, \cdots, 2N) \ . \tag{13}$$

Substituting this $\hat{G}_{(r_1\cdots r_n)}(\boldsymbol{y})$ into (10) yields

$$\dot{\boldsymbol{\phi}}(\boldsymbol{y}) \approx A\boldsymbol{\phi}(\boldsymbol{y}) + b \tag{14}$$

where

$$[A_{\alpha\ \beta}] = [C_{(q_1\cdots q_n)}^{(r_1\cdots r_n)}] \in R^{((2N+1)^n-1)\times((2N+1)^n-1)} \ ,$$

$$[b_\alpha] = [C_{(0\cdots0)}^{(r_1\cdots r_n)}] \in R^{(2N+1)^n-1}, \ \beta = \beta(q_1, \cdots, q_n).$$

Thus a formal linear state differential equation is derived by

$$\Sigma_2 : \dot{\boldsymbol{z}}(t) = A\boldsymbol{z}(t) + b \,, \tag{15}$$
$$\boldsymbol{z}(0) = \boldsymbol{\phi}\big(\boldsymbol{y}(0)\big) = \boldsymbol{\phi}\big(\pi P^{-1}(\boldsymbol{x}(0) - L) + \pi I\big) \,.$$

From (4) and (9), the inversion is carried out as follows. Let us introduce a vector $\psi_{r_i i}$ which extracts the (r_i i)-th element

$$\psi_{r_i \; i} = [0 \cdots 0 \; 1 \; 0 \cdots 0] \tag{16}$$

namely,

$$T_{r_i}(y_i) = \psi_{r_i i}\boldsymbol{\phi}(\boldsymbol{y}).$$

Since $T_1(y_i) = \cos(y_i)$, $T_2(y_i) = \sin(y_i)$, and

$$y_i = \begin{cases} \cos^{-1} T_1(y_i) \,, & \text{if } \; T_2(y_i) \geq 0 \\ 2\pi - \cos^{-1} T_1(y_i) \,, & \text{if } \; T_2(y_i) < 0 \end{cases} ,$$

the approximates $\hat{y}_i$ by the formal linearization are determined by

$$\hat{y}_i(t) = \begin{cases} \cos^{-1} \psi_{1i}\boldsymbol{z}(t) \,, & \text{if } \; \psi_{2i}\boldsymbol{z}(t) \geq 0 \\ 2\pi - \cos^{-1} \psi_{1i}\boldsymbol{z}(t) \,, & \text{if } \; \psi_{2i}\boldsymbol{z}(t) < 0 \end{cases} ,$$

and the estimate $\hat{\boldsymbol{x}}$ is obtained as

$$\hat{\boldsymbol{x}}(t) = P\Big(\frac{1}{\pi}\hat{\boldsymbol{y}}(t) - I\Big) + L. \tag{17}$$

B. Formal Linearization for Measurement Equation

In order to improve performance of estimation, an augmented measurement vector [11] is introduced which is an M-th order measurement vector $\boldsymbol{Y}(\cdot) = \boldsymbol{Y}(\boldsymbol{\eta}(\cdot))$defined as

$$\begin{aligned} \boldsymbol{Y} &= [Y_1, Y_2, \cdots, Y_i, \cdots, Y_{(M+1)^\ell - 1}]^T \\ &= [T'_{(10\cdots0)}(\boldsymbol{\eta}), T'_{(01\cdots0)}(\boldsymbol{\eta}), \cdots, T'_{(0\cdots01)}(\boldsymbol{\eta}), \\ &\quad T'_{(11\cdots0)}(\boldsymbol{\eta}), T'_{(101\cdots0)}(\boldsymbol{\eta}), \cdots, T'_{(10\cdots1)}(\boldsymbol{\eta}), \\ &\quad T'_{(20\cdots0)}(\boldsymbol{\eta}), T'_{(02\cdots0)}(\boldsymbol{\eta}), \cdots, T'_{(r_1\cdots r_\ell)}(\boldsymbol{\eta}), \\ &\quad \cdots, T'_{(M\cdots M)}(\boldsymbol{\eta})]^T \end{aligned} \tag{18}$$

where

$$T'_{(r_1\cdots r_\ell)}(\boldsymbol{\eta}) = \prod_{i=1}^{\ell} \eta_i^{r_i} \,.$$

From the given measurement equation (2), each element function of (18) is written as

$$Y_{\alpha'}(\boldsymbol{\eta}) = T'_{(r_1\cdots r_\ell)}(\boldsymbol{\eta}) = \prod_{i=1}^{\ell} \eta_i^{r_i}$$

$$= h_1^{r_1}\Big(P(\frac{\boldsymbol{y}(t)}{\pi} - I) + L\Big) h_2^{r_2}\Big(P(\frac{\boldsymbol{y}(t)}{\pi} - I) + L\Big) \cdots$$
$$h_\ell^{r_\ell}\Big(P(\frac{\boldsymbol{y}(t)}{\pi} - I) + L\Big) \equiv G'_{(r_1\cdots r_\ell)}(\boldsymbol{y}), \tag{19}$$
$$\alpha' = \alpha'(r_1, \cdots, r_\ell).$$

Applying the discrete Fourier expansion which is considered up to the N-th order to this augmented measurement equation, this $G'_{(r_1\cdots r_\ell)}(\boldsymbol{y})$ is approximated by the trigonometric functions as

$$\hat{G}'_{(r_1\cdots r_n)}(\boldsymbol{y}) \approx \sum_{q_1=0}^{2N} \cdots \sum_{q_n=0}^{2N} C'^{(r_1\cdots r_n)}_{(q_1\cdots q_n)} T_{(q_1\cdots q_n)}(\boldsymbol{y}) \tag{20}$$

where the coefficients of the discrete Fourier expansion $C'^{(r_1\cdots r_n)}_{(q_1\cdots q_n)}$ is computed in the same way as in (12)

$$C'^{(r_1\cdots r_n)}_{(q_1\cdots q_n)} \equiv \frac{2^{n-\gamma'}}{\prod_{i=1}^{n}(2N+1)} \sum_{j_1=0}^{N} \sum_{j_2=0}^{N} \cdots$$
$$\sum_{j_n=0}^{N} G'_{(r_1\cdots r_n)}(y_{1j_1}, y_{2j_2}, \cdots, y_{nj_n})$$
$$\times T_{q_1}(y_{1j_1}) T_{q_2}(y_{2j_2}) \cdots T_{q_n}(y_{nj_n}) \,, \tag{21}$$
$$\gamma' = \{\text{the number of } q_i = 0 : 1 \leq i \leq n\}.$$

Substituting this $G'_{(r_1\cdots r_\ell)}(\boldsymbol{y})$ into (19), the augmented measurement equation becomes

$$\boldsymbol{Y} \approx D\boldsymbol{\phi}(\boldsymbol{y}) + e \tag{22}$$

where

$$[D_{\alpha' \; \beta'}] = [C'^{(r_1\cdots r_\ell)}_{(q_1\cdots q_n)}] \in R^{((M+1)^\ell - 1)\times((2N+1)^n - 1)} \,,$$
$$[e_{\alpha'}] = [C'^{(r_1\cdots r_\ell)}_{(0\cdots 0)}] \in R^{((M+1)^\ell - 1)},$$
$$\beta' = \beta'(q_1, \cdots, q_n).$$

Thus a formal linear measurement equation is derived by

$$\boldsymbol{Y}(t) = D\boldsymbol{z}(t) + e \,. \tag{23}$$

C. Design of Nonlinear Observer

To the above linearized system ((15) and (23)), a linear theory is applied so that the identity observer [13] is synthesized as

$$\dot{\hat{\boldsymbol{z}}}(t) = A\hat{\boldsymbol{z}}(t) + b + K(t)\big(\boldsymbol{Y}(t) - (D\hat{\boldsymbol{z}}(t) + e)\big), \tag{24}$$
$$\hat{\boldsymbol{z}}(0) = \boldsymbol{\phi}(\hat{\boldsymbol{y}}(0)) = \boldsymbol{\phi}\big(\pi P^{-1}(\hat{\boldsymbol{x}}(0) - L) + \pi I\big)$$

where $\hat{\boldsymbol{x}}(0)$ is an initial value of the observer, $K(t)$ is an observer gain as

$$K(t) = \frac{1}{2}R(t)D^T W(t) \in R^{\left((2N+1)^n - 1\right)\times\left((M+1)^\ell - 1\right)}.$$

$R(t)$ satisfies the matrix Riccati differential equation as

$$\dot{R}(t) = AR(t) + R(t)A^T + Q(t) - R(t)D^T W(t) D R(t) \tag{25}$$

where $Q(t)$, $W(t)$ and $R(0)$ are chosen to be arbitrary real, symmetric, and positive definite matrices. With the reference to the exponential estimator [13], the error in the state estimate $\boldsymbol{e} = \boldsymbol{z} - \hat{\boldsymbol{z}}$ is uniformly asymptotically stable in the sense of Lyapunov.

From (17), the estimate $\hat{\boldsymbol{x}}(t)$ is obtained by

$$\hat{y}_i(t) = \begin{cases} \cos^{-1} \psi_{1i}\hat{\boldsymbol{z}}(t) \,, & \text{if } \; \psi_{2i}\hat{\boldsymbol{z}}(t) \geq 0 \\ 2\pi - \cos^{-1} \psi_{1i}\hat{\boldsymbol{z}}(t) \,, & \text{if } \; \psi_{2i}\hat{\boldsymbol{z}}(t) < 0 \end{cases} ,$$

and

$$\hat{\boldsymbol{x}}(t) = P\Big(\frac{\hat{\boldsymbol{y}}(t)}{\pi} - I\Big) + L. \tag{26}$$

IV. Numerical Experiments

To investigate the performance of the proposed method, numerical experiments of nonlinear observer are illustrated for a simple scalar system compared with conventional linearization based on the Taylor expansion truncated at the first order.

Suppose a dynamic scalar system

$$\dot{x} = x^2,\ x(0) = -1,\ \mathcal{D} = [-1, 0] \subset R, \qquad (27)$$

and a measurement equation

$$\eta = \sin x. \qquad (28)$$

To apply the above formal linearization, the values for changing state variable in (4) are set as

$$L = -0.1, P = 0.901,$$

the linearization function and the augmented measurement vector are

$$\boldsymbol{\phi} = \begin{pmatrix} \cos y \\ \sin y \\ \cos 2y \\ \sin 2y \\ \cos 3y \\ \sin 3y \\ \cos 4y \\ \sin 4y \\ \cos 5y \\ \sin 5y \end{pmatrix}, \boldsymbol{Y} = \begin{pmatrix} \eta \\ \eta^2 \\ \eta^3 \\ \eta^4 \\ \eta^5 \end{pmatrix}$$

respectively, when the order of the linearization function and the measurement vector M are $N = M = 5$.

To synthesize the nonlinear observer for the given system ((27) and (28)), parameters for the nonlinear observer (24) are set by the unknown value $x(0) = -1$, an initial value of the observer $\hat{\hat{x}}(0) = -0.3$,

$$Q(t) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} = diag(1,1,1,1,1),$$

$$R(0) = diag(1,1,1,1,1)$$

when $N = 5$. Besides, $W(t) = diag(10^3, 10^3, 10^3, 10^3, 10^3)$ when $M = 5$, $W(t) = diag(10^3, 10^3, 10^3, 10^3)$ when $M = 4$, $W(t) = diag(10^3, 10^3, 10^3)$ when $M = 3$, $W(t) = diag(10^3, 10^3)$ when $M = 2$, and $W(t) = 10^3$ when $M = 1$.

Fig. 1 shows the true value x of (27) and the estimates $\hat{\hat{x}}$ of (26) when N is fixed at 5 and M is varied from M = 1 to 5. *Taylor* refers to a result obtained by the conventional first order method, which is based on the Taylor expansion truncated at the first order [14]:

$$\dot{\hat{x}} = 2\hat{x}_0 x - 2\hat{x}_0^2 + \hat{x}_0$$

and

$$\eta = (\cos\hat{x}_0)x - \hat{x}_0\cos\hat{x}_0 + \sin\hat{x}_0 .$$

Here, the operating point is $\hat{x}_0$=0.6, the parameters for this observer are $Q(t) = 1$, $W(t) = 10^3$, and $R(0) = 1$.

Fig. 2 depicts the integral square errors of estimation

$$J(t) = \int_0^t \left(x(\tau) - \hat{\hat{x}}(\tau)\right)^2 d\tau$$

for the various orders and the conventional first order method(*Taylor*).

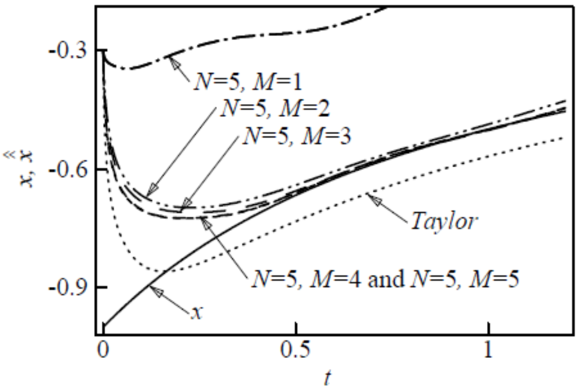

Figure 1. True value and estimates by various orders.

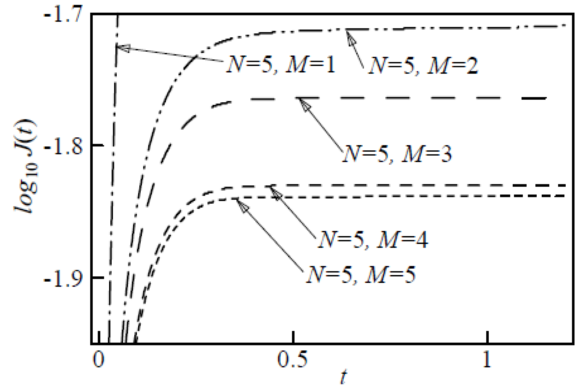

Figure 2. Integral square errors of estimation by various orders.

V. Conclusions

We have developed an observer for a nonlinear system by a formal linearization method exploiting the discrete Fourier expansion and an augmented measurement vector. By this method, coefficients of linearized systems are obtained by calculating simple summations of finite terms and sample points. It causes reducing computational complexity. Numerical experiments show that our method is better than the conventional first order method and the accuracy is improved as the orders of the augmented measurement vector increases.

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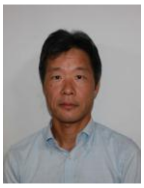

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