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Tuning and Control of Multi Variable Systems

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***Abstract*—The reported method of Equivalent transfer function(ETF) method for PI/PID decoupled controller design of multi-input multi-output square systems (Xiong, *et al.*, 2007) is extended to non-square systems. This method is applied by simulation to Example considered by Ogunnaike and Ray (1994) given by 2×3 system. Simulation studies have been carried out for servo problem and regulatory problems. Robust performance (10% increase in each process gain, 10% increase in each time delay, and 10% decrease in each time constant) of servo problem and regulatory problem are also checked. The improvement of performance of non-square controller compared with that square controller is evaluated. The performance is evaluated in terms of ISE.**

***Index Terms*—ETF, decoupled controller, servo, non square, ISE**

I. Introduction

Most of the large and complex industrial processes are naturally multi-input multi-output (MIMO) systems. Processes with unequal number of input variables (manipulated variables) and output variables (controlled variables) often arise in many industries. These systems are known as non-square systems. Such a systems may have either more outputs than inputs or more inputs than outputs. Some examples, for non-square systems with more inputs than outputs, are mixing tank process (Reeves, *et al.* 1989) 2×3 system [2], shell control problem (Vlachos et al. 1999) 7×5 system [3], etc. A common approach towards the control of non-square processes is to first square up or to square down the system through the addition or removal of appropriate inputs or outputs in order to obtain a square system. But none of the alternates is desirable. Adding unnecessary outputs to be measured can be costly, while deleting inputs leaves fewer variables to be automatically manipulated in achieving the desired control. This may result in excessive variations in the manipulated variable. Similarly, reducing the number of measured outputs decreases the amount of feedback information available to the system, and arbitrarily adding new manipulated inputs can incur unnecessary cost. Hence superior performance can be achieved by the original non-square system.

Two PI/PID based control schemes: multi-loop control and decoupling control. In multi-loop control, the multi-input multi-output (MIMO) processes are treated as a collection of multi-single loops, and a controller is designed and implemented on each loop by taking loop interactions into account. The multi-loop controller design method may fails to give acceptable responses if there exist severe loop interactions. For multi-input multi-output (MIMO) processes with severe loop interactions, the decoupling control schemes are often preferred. The decoupling control usually requires two steps: (1) design of the de-coupler to minimize the interactions among loops; and (2) design of the main loop controllers for overall system performance [5].

II. Equivalent Transfer Function (ETF) Method for PI/PID Decoupling Controller Design

Equivalent transfer function method for PI/PID decoupled controller design of multi-input multi-output systems[1] include three steps: (1) using the concepts of energy transmission ratio to obtain the effective relative gain, relative gain and relative frequency of a given transfer function matrix; (2) using the information obtained in the first step to obtain an equivalent transfer function matrix for closed loop system; and (3) designing the off-diagonal controllers based on interaction analysis and the diagonal controllers for original transfer functions of main loops.

A. General Formulation of Multi-Input Multi-Output Control

Consider an open loop stable multivariable system with n-inputs and n-outputs as shown in Fig. 1, where r_i, i = 1, 2, ..., n are the reference inputs; u_i, i = 1, 2, ..., n are the manipulated variables; y_i, i = 1, 2, ..., n are the system outputs,

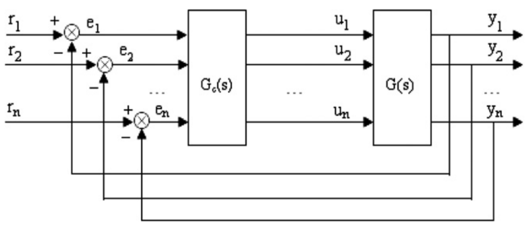

Figure 1. Closed loop multivariable control system.

$G(s)$ and $G_c(s)$ are process transfer function matrix and full dimensional controller matrix expressed by Eq. (1) and Eq. (2) respectively.

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$$G(s)=\begin{bmatrix} g_{11}(s) & g_{12}(s) & \ldots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \ldots & g_{2n}(s) \\ \ldots & \ldots & \ldots & \ldots \\ g_{n1}(s) & g_{n2}(s) & \ldots & g_{nn}(s) \end{bmatrix} \tag{1}$$

$$G_c(s)=\begin{bmatrix} g_{c,11}(s) & g_{c,12}(s) & \ldots & g_{c,1n}(s) \\ g_{c,21}(s) & g_{c,22}(s) & \ldots & g_{c,2n}(s) \\ \ldots & \ldots & \ldots & \ldots \\ g_{c,n1}(s) & g_{c,n2}(s) & \ldots & g_{c,nn}(s) \end{bmatrix} \tag{2}$$

Process transfer function is considered as a second order pulse time delay system

$$g_{ij}(s)=\frac{g_{ij}(0)}{a_{2,ij}s^2+a_{1,ij}s+1}e^{-l_{ij}s} \tag{3}$$

B. Dynamic Relative Gain Array

When a MIMO control system is closed, there exist interactions among loops as a result of the existence of non-zero off-diagonal elements in the transfer function matrix. The interactions can be dynamically measured by the dynamic relative gain defined by

$$\lambda_{ij}(s)=\frac{g_{ij}(s)}{\hat{g}_{ij}(s)} \tag{4}$$

where $\hat{g}_{ij}(s)$ is the equivalent close loop transfer function of $g_{ij}(s)$ when all other loops are closed. For overall system, the above equation can be written in a matrix form which results in the dynamic relative gain array (DRGA),

$$\lambda(s)=\begin{bmatrix} \lambda_{11}(s) & \lambda_{12}(s) & \ldots & \lambda_{1n}(s) \\ \lambda_{21}(s) & \lambda_{22}(s) & \ldots & \lambda_{2n}(s) \\ \ldots & \ldots & \ldots & \ldots \\ \lambda_{n1}(s) & \lambda_{n2}(s) & \ldots & \lambda_{nn}(s) \end{bmatrix} \tag{5}$$

Substituting Eq. (4) into (5) results in

$$\lambda(s)=\begin{bmatrix} \frac{g_{11}(s)}{\hat{g}_{11}(s)} & \frac{g_{12}(s)}{\hat{g}_{12}(s)} & \ldots & \frac{g_{1n}(s)}{\hat{g}_{1n}(s)} \\ \frac{g_{21}(s)}{\hat{g}_{21}(s)} & \frac{g_{22}(s)}{\hat{g}_{22}(s)} & \ldots & \frac{g_{2n}(s)}{\hat{g}_{2n}(s)} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{g_{n1}(s)}{\hat{g}_{n1}(s)} & \frac{g_{n2}(s)}{\hat{g}_{n2}(s)} & \ldots & \frac{g_{nn}(s)}{\hat{g}_{nn}(s)} \end{bmatrix}$$

$$=\begin{bmatrix} g_{11}(s) & g_{12}(s) & \ldots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \ldots & g_{2n}(s) \\ \ldots & \ldots & \ldots & \ldots \\ g_{n1}(s) & g_{n2}(s) & \ldots & g_{nn}(s) \end{bmatrix} \otimes \tag{6}$$

$$\begin{bmatrix} \frac{1}{\hat{g}_{11}(s)} & \frac{1}{\hat{g}_{12}(s)} & \ldots & \frac{1}{\hat{g}_{1n}(s)} \\ \frac{1}{\hat{g}_{21}(s)} & \frac{1}{\hat{g}_{22}(s)} & \ldots & \frac{1}{\hat{g}_{2n}(s)} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{1}{\hat{g}_{n1}(s)} & \frac{1}{\hat{g}_{n2}(s)} & \ldots & \frac{1}{\hat{g}_{nn}(s)} \end{bmatrix} \tag{7}$$

where the operator $\otimes$ is the hadamard product. Since $\hat{g}_{ij}(s)$ is controller dependant, it is impossible to compute K(s) without first knowing the controller parameters. By assuming the process is under perfect control, however, a simple computational algorithm for K(s) can be obtained to calculate the relative gains at each frequency point;

$$\lambda(s)=G(s)\times G^{-T}(s) \tag{8}$$

$$=\begin{bmatrix} g_{11}(s) & g_{12}(s) & \ldots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \ldots & g_{2n}(s) \\ \ldots & \ldots & \ldots & \ldots \\ g_{n1}(s) & g_{n2}(s) & \ldots & g_{nn}(s) \end{bmatrix} \otimes$$

$$\begin{bmatrix} g_{11}(s) & g_{12}(s) & \ldots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \ldots & g_{2n}(s) \\ \ldots & \ldots & \ldots & \ldots \\ g_{n1}(s) & g_{n2}(s) & \ldots & g_{nn}(s) \end{bmatrix}^{-T} \tag{9}$$

C. Energy Transmission Ratio

Express the energy transmission ratio of $g_{ij}(s)$ as

$$e_{ij}=k_{ij}\int_0^{\omega_{c,ij}}\left|g^0_{ij}(j\omega)\right|d\omega \tag{10}$$

We approximate the integration of e_{ij} by a rectangle area, i.e.,

$$e_{ij}\approx g_{ij}(0)\omega_{c,ij}\text{, i, j=1, 2, …, n} \tag{11}$$

where ω_{cij} is the critical frequency of the transfer function $g_{ij}(s)$ which can be defined in two ways:

1) $\omega_{cij}=\omega_{bij}$, where ω_{bij} for i,j = 1, 2, ..., n is the bandwidth of the transfer function $g^0_{ij}(j\omega)$ and determined by the frequency where the magnitude plot of frequency response reduced to 0.707 time, i.e.,

$$\left|g_{ij}(j\omega_{b,ij})\right|=0.707\left|g_{ij}(0)\right| \tag{12}$$

2) $\omega_{cij}=\omega_{uij}$, where ω_{uij} for i, j = 1, 2, ..., n is the ultimate frequency of the transfer function $g^0_{ij}(j\omega)$ and determined by the frequency where the phase plot of frequency response across $-\Pi$, i.e.,

$$\arg\left[g_{ij}(j\omega_{u,ij})\right] = -\Pi \quad (13)$$

For transfer function matrices with some elements without phase crossover frequencies, such as first order or second order without time delay, it is necessary to use corresponding bandwidths as critical frequencies to calculate e_{ij}. However, it is worth to point out that the phase cross over frequency information, i.e., ultimate frequency (ω_{uij}) is recommended if applicable for calculation of e_{ij}, since it is closely linked to system dynamic performance and control system design. we will use ω_{uij} as the bases for the following development.

For the frequency response of $g_{ij}(j\omega)$, e_{ij} is the area covered by $g_{ij}(j\omega)$ up to ω_{uij} . Since $|\ g^{0}_{ij}(j\omega)\ |$ represents the magnitude of the transfer function at various frequencies, e_{ij} is considered to be the energy transmission ratio from the manipulated variable u_j to the controlled variable y_i.

For the overall system, the energy transmission ratio can be expressed by effective energy transmission ratio array and given by

$$E = \begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1n} \\ e_{21} & e_{22} & \cdots & e_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ e_{n1} & e_{n2} & \cdots & e_{nn} \end{bmatrix} = G(0) \otimes \Omega \quad (14)$$

where Eq. (15) and Eq. (16)are the steady state gain array and the critical frequency array, respectively.

$$G(0) = \begin{bmatrix} g_{11}(0) & g_{12}(0) & \cdots & g_{1n}(0) \\ g_{21}(0) & g_{22}(0) & \cdots & g_{2n}(0) \\ \cdots & \cdots & \cdots & \cdots \\ g_{n1}(0) & g_{n2}(0) & \cdots & g_{nn}(0) \end{bmatrix} \quad (15)$$

$$\text{and } \Omega = \begin{bmatrix} \omega_{c,11}(s) & \omega_{c,12}(s) & \cdots & \omega_{c,1n}(s) \\ \omega_{c,21}(s) & \omega_{c,22}(s) & \cdots & \omega_{c,2n}(s) \\ \cdots & \cdots & \cdots & \cdots \\ \omega_{c,n1}(s) & \omega_{c,n2}(s) & \cdots & \omega_{c,nn}(s) \end{bmatrix} \quad (16)$$

D. *Effective Relative Gain Array*

The effective relative gain, ϕ_{ij} , between output variable y_i and input variable u_j is define as the ratio of two effective energy transmission ratio:

$$\phi_{ij} = \frac{e_{ij}}{\hat{e}_{ij}} \quad (17)$$

where $\hat{e}_{ij}$ is the effective energy transmission ratio between output variable y_i and input variable u_j when all other loops are closed. When the effective relative gains are calculated for all the input/output combinations of a multivariable process, it results in an array, ERGA, which can be calculated by

$$\phi = E \otimes E^{-T} = \begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{bmatrix} \quad (18)$$

The ERGA based loop pairing rules requires that manipulated and controlled variables in the main loop be paired by those pairs whose ERGA values are positive and closest to 1.0.

E. *Relative Frequency Array*

According to Effective relative gain array, Place the qualified pairs to the diagonal position and rewrite Eq. (17) as

$$\hat{g}_{ij}(0)\hat{\omega}_{c,ij} = \frac{g_{ij}(0)\omega_{c,ij}}{\phi_{ij}} \quad (19)$$

Since

$$\hat{g}_{ij}(0) = \frac{g_{ij}(0)}{\lambda_{ij}} \quad (20)$$

Which can be calculated by Eq. (8) with s = 0,

$$\lambda = G \otimes G^{-T}(0) = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \lambda_{n1} & \lambda_{n2} & \cdots & \lambda_{nn} \end{bmatrix} \quad (21)$$

We obtain

$$\frac{\phi_{ij}}{\lambda_{ij}} = \frac{\omega_{c,ij}}{\hat{\omega}_{c,ij}} \equiv \gamma_{ij} \quad (22)$$

Which can also be expressed in matrix form, i.e., relative frequency array (RFA), and calculated by

$$\gamma = \phi \odot \lambda = \begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{bmatrix} \odot \begin{bmatrix} \lambda_{11}(s) & \lambda_{12}(s) & \cdots & \lambda_{1n}(s) \\ \lambda_{21}(s) & \lambda_{22}(s) & \cdots & \lambda_{2n}(s) \\ \cdots & \cdots & \cdots & \cdots \\ \lambda_{n1}(s) & \lambda_{n2}(s) & \cdots & \lambda_{nn}(s) \end{bmatrix} \quad (23)$$

where $\odot$ is the hadamard division, λ_{ij} and γ_{ij} are the steady state relative gain and relative critical frequency of loop i - j, respectively.

F. Equivalent Transfer Function of Closed Loop System and Parameterization of Controllers

Using λ_{ij} and γ_{ij}, we can now determine the equivalent transfer function $\hat{g}_{ij}(s)$. Rewrite $g_{ij}(j\omega)$ as

$$g_{ij}(s) = g_{ij}(0)e^{-l_{ij}s}g_{ij}^{0}(s) \quad (24)$$

where $g_{ij}(0)$, l_{ij} and $g_{ij}^{0}(s)$ are steady state gain, time delay and normalized transfer function of $g_{ij}(s)$ excluding time delay, i.e., $g_{ij}^{0}(0) = 1$, respectively. As control loop transfer functions when other loops closed will have similar frequency properties with when other loops open if it is well paired , we can let the effective transfer functions have the same structures as the corresponding open loop transfer functions but with two different parameters, i.e.,

$$g^{\wedge}_{ij}(s) = g^{\wedge}_{ij}(0)e^{-l^{\wedge}_{ij}s}g_{ij}^{0}(s) \quad (25)$$

In Eq. (24), $\hat{g}_{ij}(0)$ reflects the gain change and can be determined by Eq. (20), while $\hat{l}_{ij}$ reflects the change in critical frequency. Although the critical frequency is generally be affected by both time constant and time delay, they are exchangeable by linear approximation, it is reasonable to change only time delay to reflect the phase changes in the low frequency range which is given by

$$l^{\wedge}_{ij} = \frac{\omega_{c,ij}}{\omega_{c,ij}^{\wedge}}l_{ij} \equiv \gamma_{ij}l_{ij} \quad (26)$$

The design of full dimensional PI/PID controller consists of two parts:

1) Off-diagonal controllers: The main task of the off diagonal controllers is to minimize the interactions among loops.
2) Diagonal controllers: The diagonal controllers are to provide the desired performance of the closed loop control system.

We use the gain and phase margins approach to design the controller.

Theoretically, any SISO controller design approach can be employed. This is because the interaction is already approximately considered into the equivalent transfer functions. The gain and phase margins approach is selected because it provides good performance in terms of robustness with respect to the uncertainties in both process model and disturbance, and might be more acceptable by process control engineers.

The standard PID controller is adopted as

$$g_{c,ij}(s) = k_{p,ji} + \frac{1}{k_{i,ji}s} + k_{d,ji}s \quad (27)$$

Rewrite Eq. (27) as

$$g_{c,ij}(s) = k_{ji}\frac{As^2 + Bs + c}{s} \quad (28)$$

where $A = k_{d,ji}/k_{ji}, B = k_{p,ji}/k_{ji} andC = k_{i,ji}/k_{ji}$ by selecting $A = a_{2,ij}, B = a_{1,ij} andc = 1$.

The forward transfer function according to Eq. (3), Eq. (25) and Eq. (28) becomes

$$g_{c,ji}(s)g_{ij}^{\wedge}(s) = k_{ji}\frac{g_{ij}^{\wedge}(0)}{s}e^{-l_{ij}^{\wedge}s} \quad (29)$$

Denoting the gain and phase margin specifications as $A_{m,ij}$ and $\varphi_{m,ij}$, and their crossover frequencies as $\omega_{g,ij}$ and $\omega_{p,ij}$, respectively, we have

$$\arg\left[g_{c,ji}(j\omega_{g,ij})g_{ij}^{\wedge}(j\omega_{g,ij})\right] = -\pi \quad (30)$$

$$A_{m,ij}\left|g_{c,ji}(j\omega_{g,ij})g_{ij}^{\wedge}(j\omega_{g,ij})\right| = 1 \quad (31)$$

$$\left|g_{c,ji}(j\omega_{p,ij})g_{ij}^{\wedge}(j\omega_{g,ij})\right| = 1 \quad (32)$$

$$\psi_{m,ij} = \pi + \arg\left[g_{c,ji}(j\omega_{p,ij})g_{ij}^{\wedge}(j\omega_{p,ij})\right] \quad (33)$$

By substitution and simplification to above equations, we obtain

$$\omega_{g,ij}l_{ij}^{\wedge} = \frac{\pi}{2} \quad (34)$$

$$A_{m,ij} = \frac{\omega_{g,ij}}{k_{ji}g_{ij}^{\wedge}(0)} \quad (35)$$

$$k_{ji}g_{ij}^{\wedge}(0) = \omega_{p,ij} \quad (36)$$

$$\psi_{m,ij} = \frac{\pi}{2} - \omega_{p,ij}l_{ij}^{\wedge} \quad (37)$$

Which results

$$\psi_{m,ij} = \frac{\pi}{2}(1 - \frac{1}{A_{m,ij}}) \quad (38)$$

$$k_{ji} = \frac{\pi}{2A_{m,ij}l_{ij}^{\wedge}g_{ij}^{\wedge}(0)} \quad (39)$$

Considering Eq. (20) and Eq. (26), the PID parameters are given by

$$\begin{bmatrix} k_{p,ji} \\ k_{i,ji} \\ k_{d,ji} \end{bmatrix} = \frac{\pi\lambda_{ij}}{2A_{m,ij}\gamma_{ij}l_{ij}g_{ij}(0)}\begin{bmatrix} a_{2,ij} \\ 1 \\ a_{1,ij} \end{bmatrix} \quad (40)$$

Once loop interactions are dealt with by the off-diagonal decoupling controllers, the diagonal loops can be considered as *n* independent loops. Each controller can thus be independently designed by single loop approaches based on the corresponding diagonal transfer functions,

$$g_{ii}(s) = \frac{g_{ii}(0)}{a_{2,ii}s^2 + a_{1,ii}s + 1} e^{-l_{ii}s} \quad (41)$$

Again, following the gain and phase margins approach, the controller parameters of main loops are given as

$$\begin{bmatrix} k_{p,ii} \\ k_{i,ii} \\ k_{d,ii} \end{bmatrix} = \frac{\pi}{2A_{m,ii} l_{ii} g_{ii}(0)} \begin{bmatrix} a_{2,ii} \\ 1 \\ a_{1,ii} \end{bmatrix} \quad (42)$$

III. Extension of ETF Method to Non-Square Systems

In the present work the Equivalent transfer function method by Xiong, *et al.*, (2007) [1] for PI/PID decoupled controller design of multi-input multi-output square systems is extended to non-square systems. This method has been applied to an example considered by Ogunnaike and Ray (1994) [4] given by 2×3 system Simulation studies have been carried out for this example for servo problem, and regulatory problems. Robust performance (10% increase in each process gain, 10% increase in each time delay, and 10% decrease in each time constant) of servo problem, and regulatory problem is also checked. The improvement of performance of non-square controller compared with that square controller is evaluated. The performance is evaluated in terms of ISE.

Example: considered by Ogunnaike and Ray, 1994. The vector form of transfer function model is

$$Y(s) = G(s) * U(s) \quad (43)$$

where

$$G(s) = \begin{bmatrix} \frac{0.5}{3s+1}e^{-0.2s} & \frac{0.07}{2.5s+1}e^{-0.3s} & \frac{0.04}{2.8s+1}e^{-0.03s} \\ \frac{0.004}{1.5s+1}e^{-0.4s} & \frac{-0.003}{s+1}e^{-0.2s} & \frac{-0.001}{1.6s+1}e^{-0.4s} \end{bmatrix} \quad (44)$$

$$Y(s) = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (45)$$

$$U(s) = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad (46)$$

The steady state gain matrix is

$$G(0) = \begin{bmatrix} 0.5 & 0.07 & 0.04 \\ 0.004 & -0.003 & -0.001 \end{bmatrix} \quad (47)$$

As inverse does not exist for non-square systems. Moore -Penrose pseudo-inverse is used for non-square systems. The pseudo-inverse of $G(0)$ is

$$[pinvG(0)] = \begin{bmatrix} 1.6637 & 41.8688 \\ 1.9603 & -247.3312 \\ 0.7736 & -90.5310 \end{bmatrix} \quad (48)$$

Transpose of pseudo-inverse of G(0) is

$$(pinvG(0))' = \begin{bmatrix} 1.6637 & 1.9603 & 0.7736 \\ 41.8688 & -247.3312 & -90.5310 \end{bmatrix} \quad (49)$$

Dynamic relative gain array (DRGA) is given by

$$DRGA = G(0) \otimes (pinvG(0))' \quad (50)$$

$$= \begin{bmatrix} 0.8318 & 0.1372 & 0.0309 \\ 0.1675 & 0.7420 & 0.0905 \end{bmatrix} \quad (51)$$

Critical frequency array (Ω) is

$$\Omega = \begin{bmatrix} 8 & 5.45 & 52 \\ 4.3 & 8.4 & 4.27 \end{bmatrix} \quad (52)$$

Effective energy transmission ratio array (E) is

$$E = G(0) \otimes \Omega \quad (53)$$

$$E = \begin{bmatrix} 4 & 0.3815 & 2.08 \\ 0.0172 & -0.0252 & -0.00427 \end{bmatrix} \quad (54)$$

Effective relative gain array (ERGA) is

$$ERGA = E \otimes E^{-T} \quad (55)$$

$$= \begin{bmatrix} 0.6938 & 0.0368 & 0.2694 \\ 0.1536 & 0.7979 & 0.0485 \end{bmatrix} \quad (56)$$

Effective relative gain array is positive and nearly equal to unity for best pairing. So y_1 is paired with u_1 and u_3 and y_2 is paired with u_2.

Relative frequency array (RFA) is

$$RFA = \frac{ERGA}{DRGA} \quad (57)$$

$$RFA = \begin{bmatrix} 0.8340 & 0.2682 & 8.7184 \\ 0.9170 & 1.0753 & 0.5359 \end{bmatrix} \quad (58)$$

The gain margin for all loops are specified as $A_{m,ij} = 50db$ Full dimensional controller matrix is obtained by using gain and phase margin approach.

Controllers are designed by gain and phase margin approach using the following controller parameters.

Parameters of main loop controllers are

$$\begin{bmatrix} k_{p,ii} \\ k_{i,ii} \\ k_{d,ii} \end{bmatrix} = \frac{\pi}{2A_{m,ii} l_{ii} g_{ii}(0)} \begin{bmatrix} a_{2,ii} \\ 1 \\ a_{1,ii} \end{bmatrix} \quad (59)$$

$$k_{p11} = 1.40 \qquad k_{i11} = 1.40/3$$

$$k_{p22} = -37.939 \qquad k_{i22} = -37.939$$

Parameters of decoupling controllers are

$$\begin{bmatrix} k_{p,ji} \\ k_{i,ji} \\ k_{d,ji} \end{bmatrix} = \frac{\pi\lambda_{ij}}{2A_{m,ij}\gamma_{ij} l_{ij} g_{ij}(0)} \begin{bmatrix} a_{2,ij} \\ 1 \\ a_{1,ij} \end{bmatrix} \quad (60)$$

$$k_{p12} = 4.2 \qquad k_{i12} = 4.2/1.5$$
$$k_{p21} = 2.089 \qquad k_{i21} = 2.089/2.5$$
$$k_{p31} = 0.1824 \qquad k_{i31} = 0.1824/2.8$$
$$k_{p32} = -13.88 \qquad k_{i32} = -13.88/1.6$$

Full dimensional controller matrix is

$$G_c(s) = \begin{bmatrix} 1.40(1+\frac{1}{3s}) & 4.2(1+\frac{1}{1.5s}) \\ 2.089(1+\frac{1}{2.5s}) & -37.939(1+\frac{1}{s}) \\ 0.1824(1+\frac{1}{2.8s}) & -13.88(1+\frac{1}{1.6s}) \end{bmatrix} \quad (61)$$

IV. Simulation Results

A. Servo Responses

Fig. 2 shows the response of y1 and interaction in y2 for unit step change in set point r1. Fig. 3 shows the response of y2 and interaction in y1 for unit step change in set point r2. Interaction in y2 due to step change in r1 is less compared to interaction in y1 due to step change in r2.

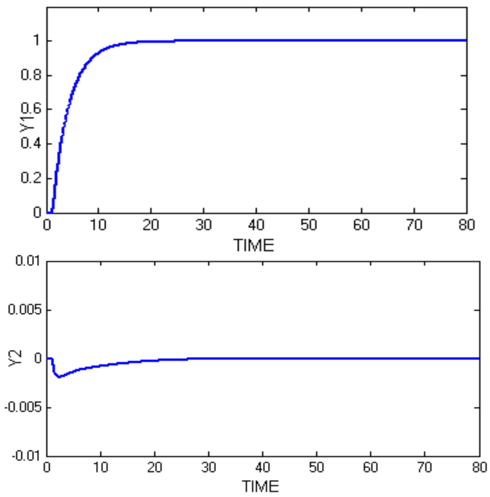


Figure 2. Response of y1 and interaction in y2 due to step change in r1

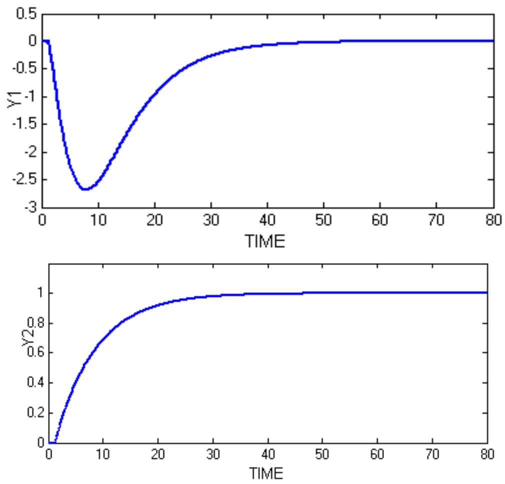


Figure 3. Interaction in y1 and response of y2 due to step change in r2

B. Regulatory responses

Fig. 4, Fig. 5 & Fig. 6 shows the responses of the designed controllers due to change in load variables d1, d2, d3 respectively. It is assumed that the load transfer function matrix is same as that of process transfer function matrix.

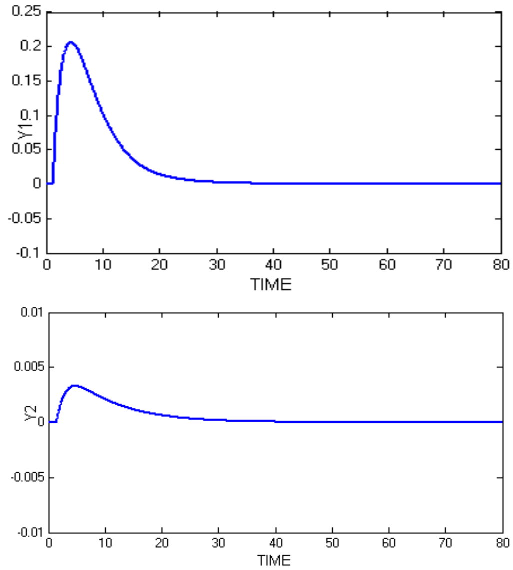


Figure 4. Response of y1 and interaction in y2 due to step change in d1

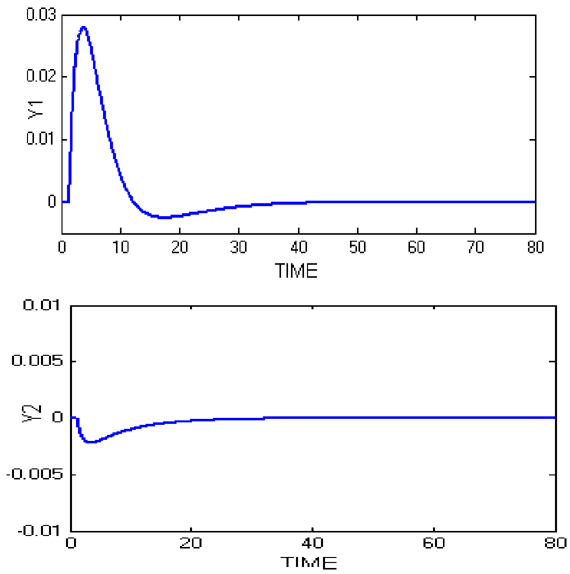
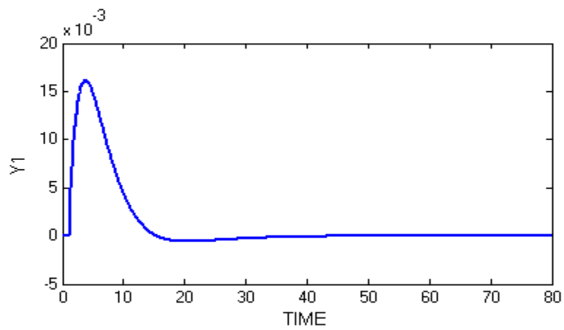


Figure 5. Response of y1 and interaction in y2 due to step change in d2



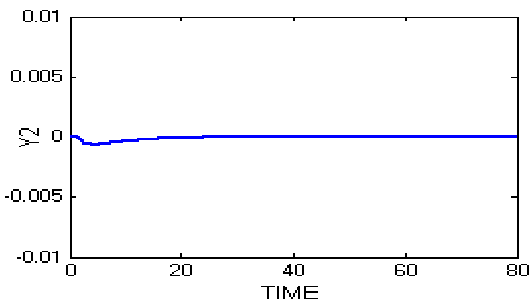


Figure 6. Response of y1 and interaction in y2 due to step change in d3

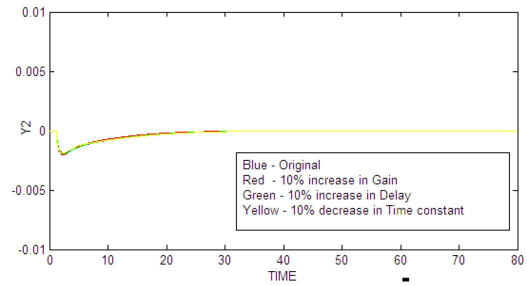


Figure 7. Comparison of response of y1 and interaction in y2 due to step change in r1 when 10% increase in gain, 10% increase in time delay and 10% decrease in time constant with original response

C. Robustness Studies

Model parameters like process gain, time delay and time constant are consider to design any control system. Once controller designed the system performance will be satisfactory in simulation but not in real time due to model mismatch. But if we design a robust controller by considering same model parameters with deviation the system performance will be satisfactory in both simulation and real time.

Robustness studies can be carried out for the perturbed system by

a. Considering the 10% deviation in the time delay.
b. Considering the 10% deviation in time constant.
c. Considering the 10% deviation in the gain.

Controllers designed by decoupling technique are giving the similar results for the predicted model and the actual plant model.

1) Servo responses

Fig. 7 shows the comparison of response of y1 and interaction in y2 for unit step change in set point r1 when 10% increase in gain, 10% increase in time delay and 10% decrease in time constant with original response. Fig. 8 shows the comparison of response of y2 and interaction in y1 for unit step change in set point r2 when 10% increase in gain, 10% increase in time delay and 10% decrease in time constant with original response. Almost all responses are similar. So decoupling controllers achieve robust performance.

2) Regulatory responses

Fig. 9, Fig. 10 & Fig. 11 shows the comparison of response of y1 and interaction in y2 for unit step change in d1, d2 and d3 respectively when 10% increase in gain, 10% increase in time delay and 10% decrease in time constant with original response.. Almost all responses are similar. So Decoupling controller achieve robust performance.

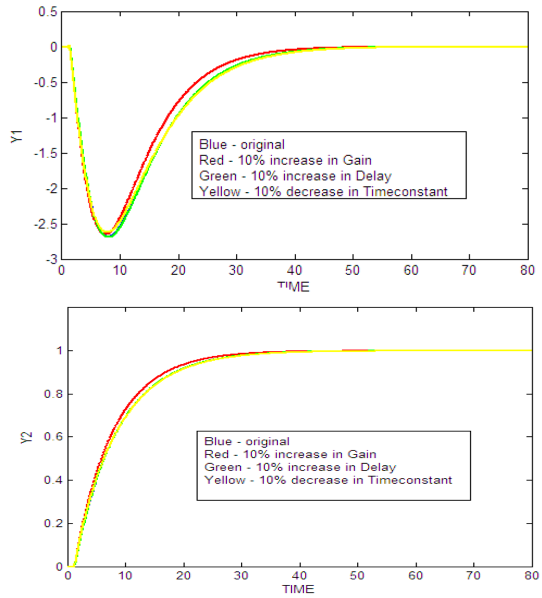


Figure 8. Comparison of interaction in y1 and response of y2 due to step change in r2 when 10% increase in gain, 10% increase in time delay and 10% decrease in time constant with original response

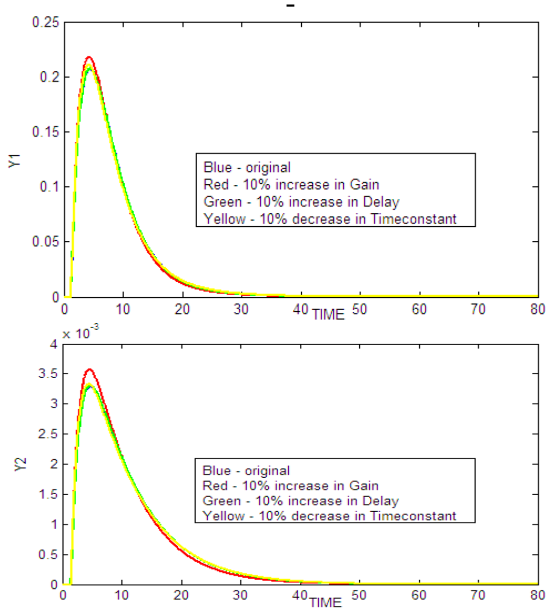
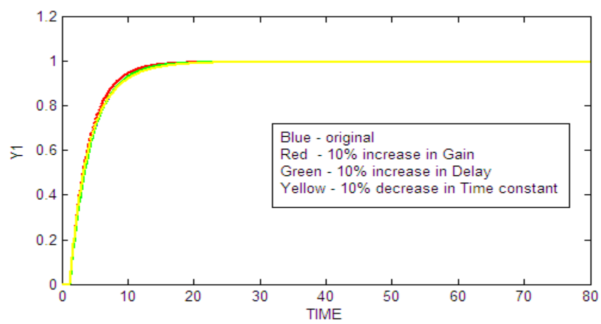


Figure 9. Comparison of response of y1 and interaction in y2 due to step change in d1 when 10% increase in gain, 10% increase in time delay and 10% decrease in time constant with original response



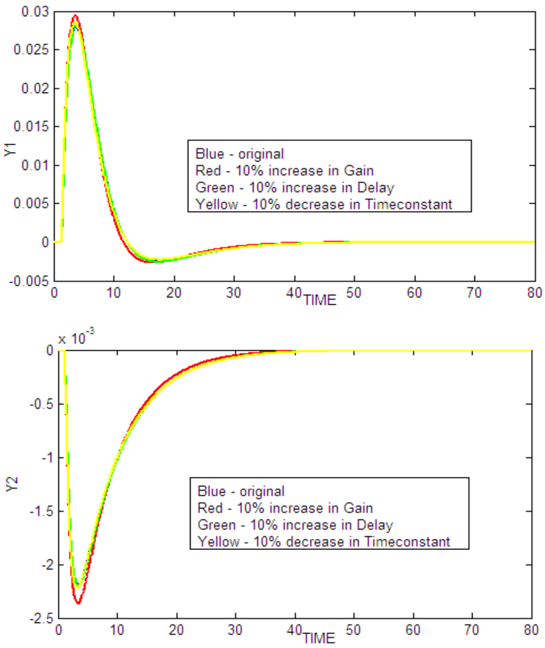


Figure 10. Comparison of response of y1 and interaction in y2 due to step change in d2 when 10% increase in gain, 10% increase in time delay and 10% decrease in time constant with original response

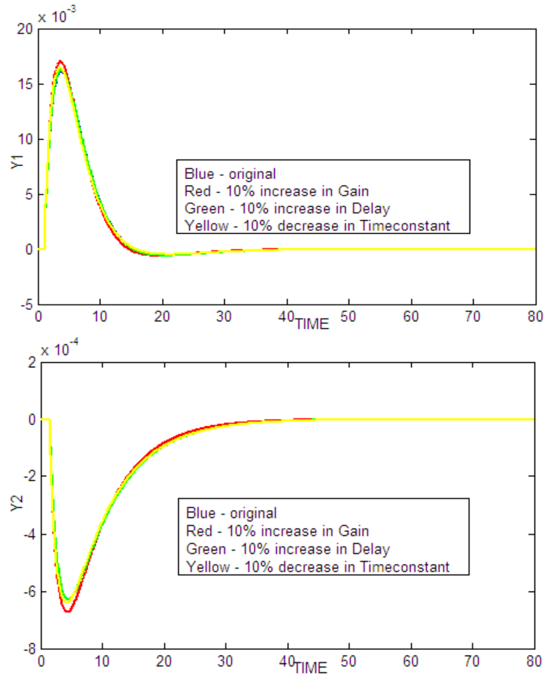


Figure 11. Comparison of response of y1 and interaction in y2 due to step change in d3 when 10% increase in gain, 10% increase in time delay and 10% decrease in time constant with original response

D. *Manipulated Variables Time Behaviour*

Fig. 12 and Fig. 13 shows the manipulated variables time behaviour due to step change in r1 and r2 respectively.

Fig. 14 and Fig. 15 shows the comparison of manipulated variables time behaviour due to step change in r1 and r2 respectively when 10% increase in gain, 10% increase in time delay and 10% decrease in time constant with original behaviour.. Almost all time behaviours are similar. So decoupling controller achieve robust performance.

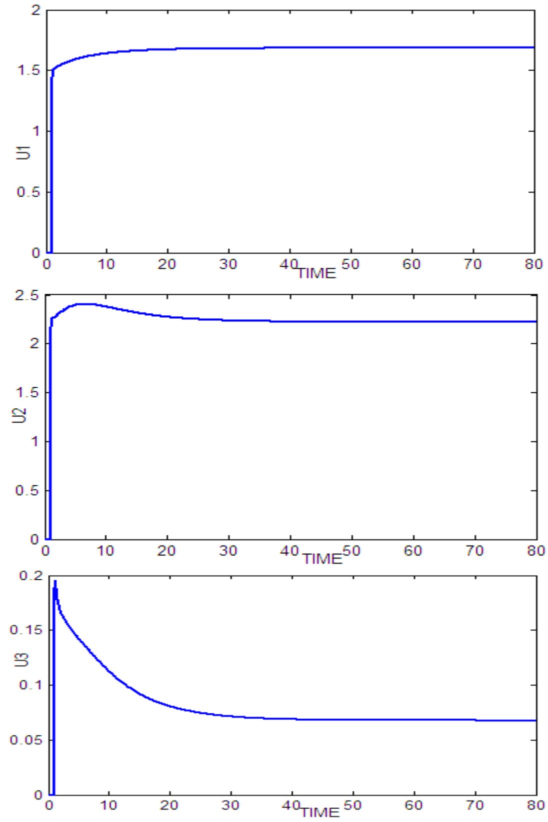


Figure 12. Manipulated variables time behaviour due to step change in r1

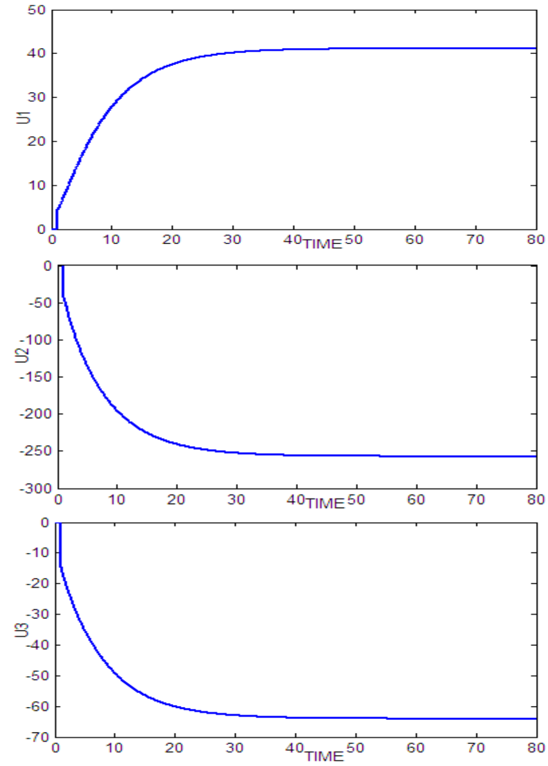


Figure 13. Manipulated variables time behaviour due to step change in r2

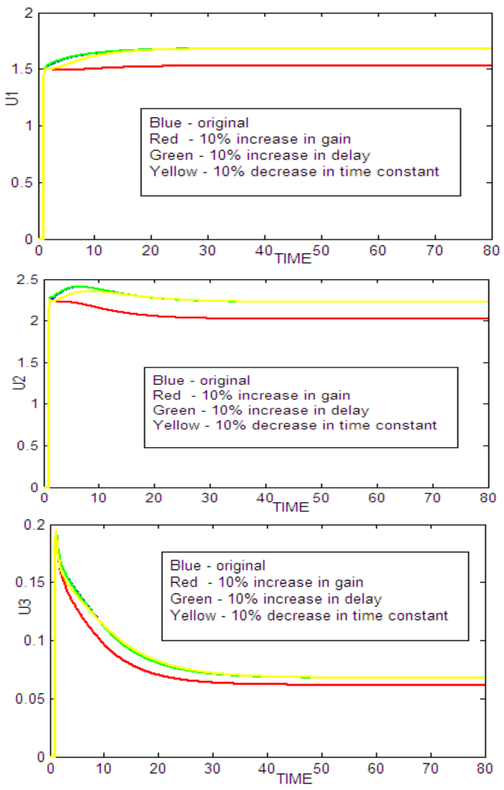


Figure 14. Comparison of manipulated variables time behaviour due to step change in r1 when 10% increase in gain, 10% increase in time delay and 10% decrease in time constant with original response

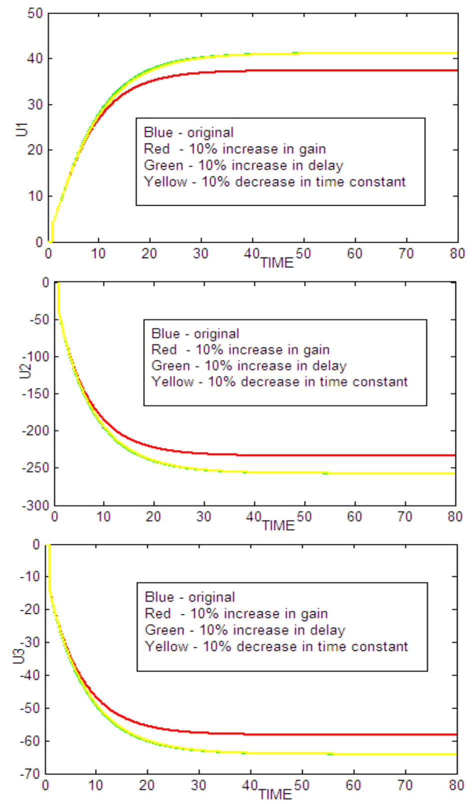


Figure 15. Comparison of manipulated variables time behaviour due to step change in r2 when 10% increase in gain, 10% increase in time delay and 10% decrease in time constant with original response

V. Comparision with Square Systems

To bring down the system to a square form, the input (u3) is neglected. The resulting system is given by

$$G(s)=\begin{bmatrix}\frac{0.5}{3s+1}e^{-0.2s} & \frac{0.07}{2.5s+1}e^{-0.3s}\\ \frac{0.004}{1.5s+1}e^{-0.4s} & \frac{-0.003}{s+1}e^{-0.2s}\end{bmatrix}$$

The steady state gain matrix is

$$G(0)=\begin{bmatrix}0.5 & 0.07\\ 0.004 & -0.003\end{bmatrix} \tag{62}$$

Dynamic relative gain array (DRGA) is given by

$$DRGA(\lambda)=G(0)\otimes(invG(0))' \tag{63}$$

$$DRGA=\begin{bmatrix}0.8427 & 0.1573\\ 0.1573 & 0.8427\end{bmatrix} \tag{64}$$

Critical frequency array (Ω) is

$$\Omega=\begin{bmatrix}8 & 5.45\\ 4.3 & 8.4\end{bmatrix} \tag{65}$$

Effective energy transmission ratio array is

$$E=G(0)\otimes\Omega \tag{66}$$

$$E=\begin{bmatrix}4 & 0.3815\\ 0.0172 & -0.0252\end{bmatrix} \tag{67}$$

Effective relative gain array (ERGA) is

$$ERGA(\phi)=E.*E^{-T} \tag{68}$$

$$ERGA=\begin{bmatrix}0.9389 & 0.0611\\ 0.0611 & 0.9389\end{bmatrix} \tag{69}$$

Effective relative gain array is positive and nearly equal to unity for best pairing. So y_1 is paired with u_1 and y_2 is paired with u_2.

Relative frequency array (RFA) is

$$RFA=\phi\div\lambda=\begin{bmatrix}1.1141 & 0.3884\\ 0.3884 & 1.1141\end{bmatrix} \tag{70}$$

The gain margin for all loops are specified as $A_{m,ij}=50db$. Full dimensional controller matrix is obtained by using gain and phase margin approach. Controllers are designed by gain and phase margin approach using the following controller parameters.

Parameters of main loop controllers are

$$\begin{bmatrix}k_{p,ii}\\ k_{i,ii}\\ k_{d,ii}\end{bmatrix}=\frac{\pi}{2A_{m,ii}l_{ii}g_{ii}(0)}\begin{bmatrix}a_{2,ii}\\ 1\\ a_{1,ii}\end{bmatrix} \tag{71}$$

$$k_{p11}=0.7 \qquad k_{i11}=0.7/3$$

$$k_{p22}=-18.9695 \qquad k_{i22}=-18.9695$$

Parameters of decoupling controllers are

$$\begin{bmatrix} k_{p,ji} \\ k_{i,ji} \\ k_{d,ji} \end{bmatrix} = \frac{\pi\lambda_{ij}}{2A_{m,ij}\gamma_{ij} l_{ij} g_{ij}(0)} \begin{bmatrix} a_{2,ij} \\ 1 \\ a_{1,ij} \end{bmatrix} \tag{72}$$

$$k_{p12} = 4.641 \qquad k_{i12} = 4.641/1.5$$

$$k_{p21} = 0.8266 \qquad k_{i21} = 0.8266/2.5$$

Full dimensional controller matrix is

$$G_c(s) = \begin{bmatrix} 0.7(1+\frac{1}{3s}) & 4.641(1+\frac{1}{1.5s}) \\ 0.8266(1+\frac{1}{2.5s}) & -18.9695(1+\frac{1}{s}) \end{bmatrix} \tag{73}$$

A. *Comparison of Servo Responses of Non-Square System with Square System*

Fig. 16 compares the response of y1 and interaction in y2 for unit step change in set point r1. Fig. 17 compares the response of y2 and interaction in y1 for unit step change in set point r2. It is clear that Settling time is less for non-square system compared to square system.

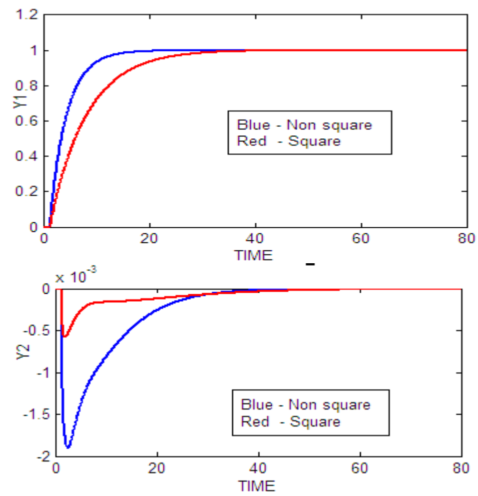


Figure 16. Comparison of response of y1 and interaction in y2 of non-square system with square system due to step change in r1

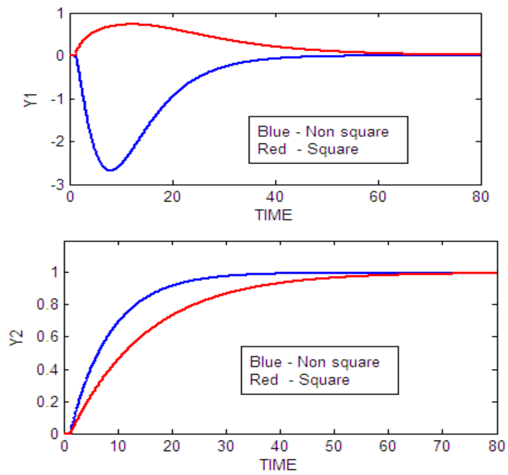


Figure 17. Comparison of interaction in y1 and response of y2 of non-square system with square system due to step change in r2

B. *Comparison of Manipulated Variables Time Behaviour of Non-Square System with Square System*

Fig. 18 compares the manipulated variables time behaviour of non-square system with square system due to step change in r1. Fig. 19 compares the manipulated variables time behaviour of non-square system with square system due to step change in r2. It is clear that Non-square system gives better manipulated variables time behaviour compared with square system.

C. *Comparison of Servo Responses of Robustness Problem of Non-Square System with Square System*

Fig. 20 compares the response of y1 and interaction in y2 for unit step change in set point r1 of perturbed non-square system with perturbed square system. Fig. 21 compares the response of y2 and interaction in y1 for unit step change in set point r2 of perturbed non-square system with perturbed square system. Non-square system gives better responses than square down the system.

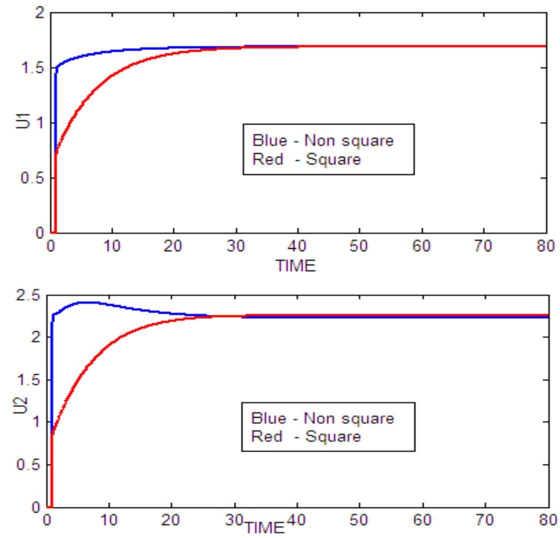


Figure 18. Comparison of manipulated variables time behaviour of non-square system with square system due to step change in r1

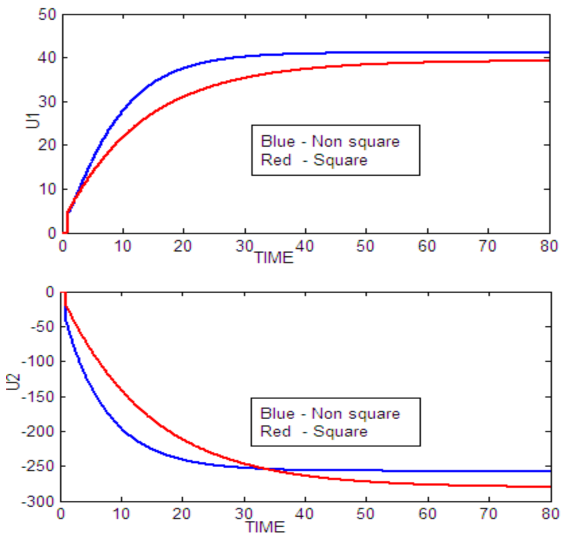


Figure 19. Comparison of manipulated variables time behaviour of non-square system with square system due to step change in r2

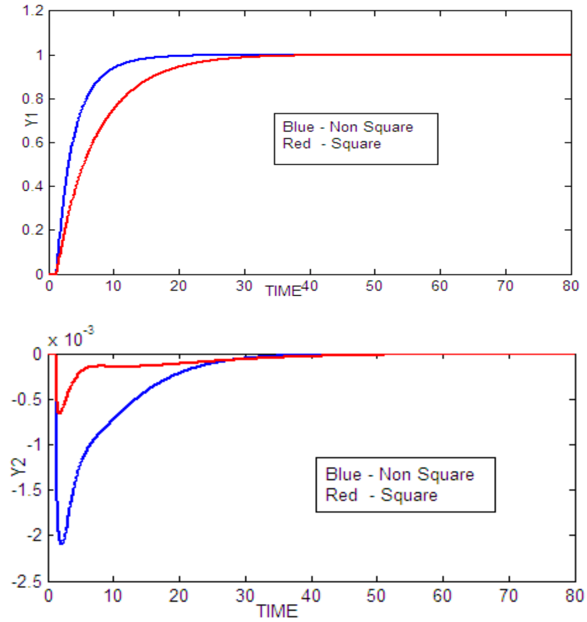


Figure 20. Comparison of response of y1 and interaction in y2 of robustness problem (10% increase in each process gain, 10% increase in each time delay and 10% decrease in each time constant) of non-square system with square system due to step change in r1

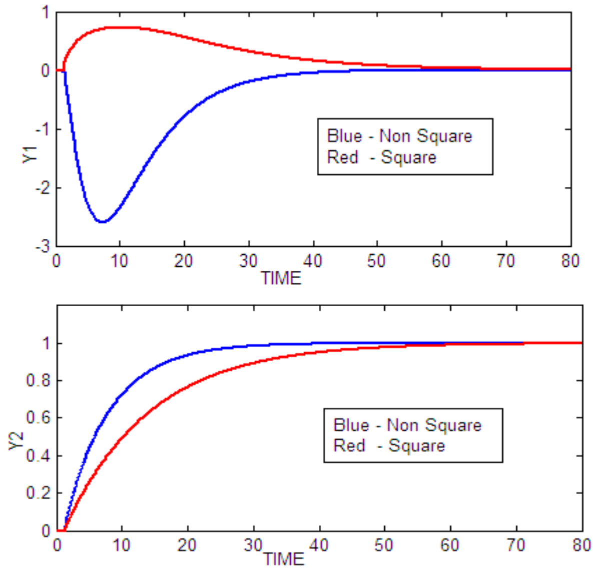


Figure 21. Comparison of interaction in y1 and response of y2 of robustness problem (10% increase in each process gain, 10% increase in each time delay and 10% decrease in each time constant) of non-square system with square system due to step change in r2

D. *Comparison Manipulated Variables Time Behavior of Robustness Problem of Non-Square System with Square System*

The manipulated variables time behaviour of non-square system with square system due to step change in r1 and r2 of perturbed system (10% increase in each process gain, 10% increase in each time delay and 10% decrease in each time constant) is compared. Non-square system gives better manipulated variables time behaviour compared with square system.

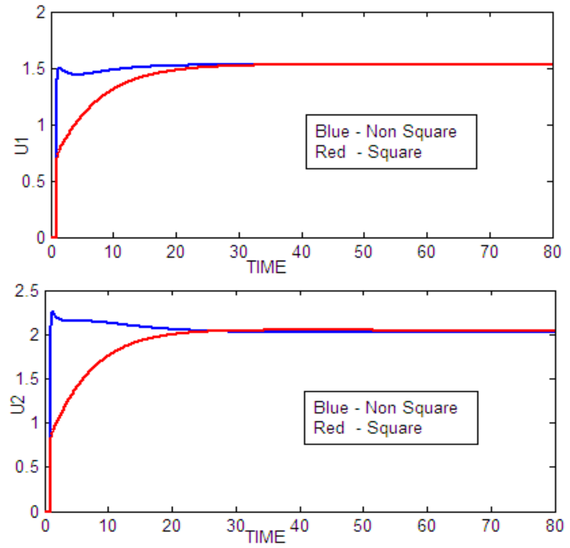


Figure 22. Comparison of manipulated variables time behaviour of robustness problem (10% increase in each process gain, 10% increase in each time delay and 10% decrease in each time constant) of non-square system with square system due to step change in r1.

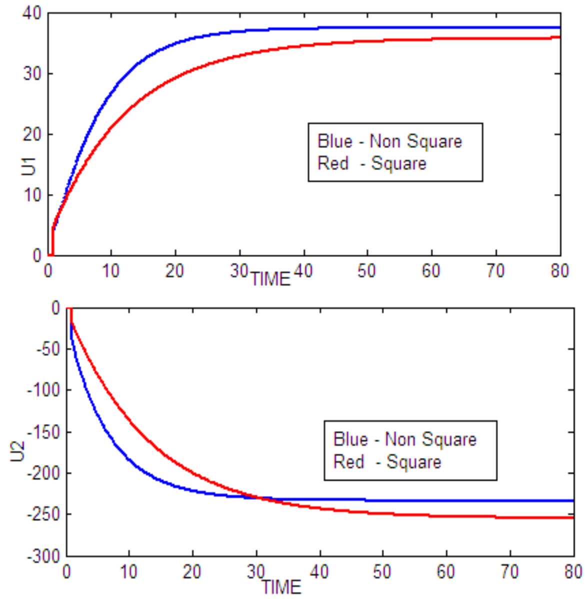


Figure 23. Comparison of manipulated variables time behaviour of robustness problem (10% increase in each process gain, 10% increase in each time delay and 10% decrease in each time constant) of non-square system with square system due to step change in r2

VI. Comparison of ISE Values of Non-Square Controller with Square Controller

Table I compares the ISE values of non-square controller with square controller. ISE values of perfect non-square model and perturbed non-square model are low compared with ISE values of perfect square model and perturbed square model. ISE values of non-square system is nearly 45% of the ISE values of the square system. So control of non-square system is better rather than squaring down the system.

TABLE I. COMPARISON OF ISE VALUES OF NON-SQUARE CONTROLLER WITH SQUARE CONTROLLER.

System	Step change in	ISE values for output y1	ISE values for output y2	Sum of ISE values
Perfect model Non-square systems	r1	2.34	0	2.34
	r2	3.29	4.555	7.845
Perfect model square systems	r1	4.97	0.49	5.46
	r2	1.23	7.936	9.166
Perturbed model Non-square systems	r1	2.46	0	2.46
	r2	3.32	4.69	8.01
Perturbed model square systems	r1	5.03	0.61	5.64
	r2	1.29	8.173	9.463

VII. CONCLUSION

Equivalent transfer function method for PI/PID decoupled controller design of multi-input multi-output square systems is extended to non-square systems. This method has been applied to an example considered by Ogunnaike and Ray (1994) given by 2×3 system. Simulation studies have been carried out for servo problem, and regulatory problems. Robust performance (10% increase in each process gain, 10% increase in each time delay, and 10% decrease in each time constant) of servo problem, and regulatory problem is also checked for the example. The improvement of performance of non-square controller compared with that square controller is evaluated. Simulation results show that non-square controllers gives better response compared with square controllers. ISE values of non-square system are nearly 45% of ISE values of square system (Example considered by Ogunnaike and Ray). So significant improvements in the performance and robustness are obtained when the non-square system is controlled in its original form rather than squaring it down.

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