Transient Stability Analysis of a Two-Machine Power System under Different Fault Clearing Times

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***Abstract*—This paper presents a new method for analyzing transient stability of a faulted two-machine system. This method uses a set of closed loop transfer function for each machine and derives the dominant root by taking Laplace transformation of the machine's nonlinear equation. A three-phase fault is considered near bus 4 on the line 4-5 and the system has been studied by the proposed method. This system is again studied by the CYME power system simulation software at different fault clearing times of 0.05, 0.07, 0.10, 0.15 and 0.225 seconds respectively, after disconnecting the line 4-5. The results obtained by the proposed method have been compared with the results obtained by simulation. It has been found that the two-machine system is stable for the first four fault clearing times while it is unstable for the fault clearing time of 0.225 seconds.**

***Index Terms*—transient stability analysis, CYME software, fault clearing time, stable, unstable second**

I. INTRODUCTION

Demand of electrical power is increasing day by day due to increasing population, modern shopping malls and industrial sectors. To meet this high demand, the power system networks are growing to be more complex system by trying to accommodate more interconnections, power generating units and extra transmission lines. In this case, maintaining transient stability of a power system network is becoming a challenging task. Transient stability is the ability of a system to maintain synchronism when it is subjected to a large disturbance within a short duration. This sudden disturbance affects the system's performance such as large variations in generator's rotor angles, power (real and reactive) flows, bus voltages and other system parameters. So far, transient stability has been studied by different methods such as time domain analysis (TDA), Lyapunov energy method and hybrid method. Time domain analysis has excellent modeling capability, but it needs large amount of computational effort [1]. The hybrid method generally combines the desirable features of time-domain analysis and direct method of transient stability analysis [2]. In this case, a special analysis is required to evaluate the transient energy function in order to derive a stability index for fast derivation of transient stability limits.

A uniform approach has been applied to the transient stability analysis of a small power system leading to much improved estimates of stability region and critical clearing times as compared to traditional energy functions, numerical energy functions and two time-scale energy functions [3]. In this study, three energy functions have been used to derive the final stability index. In [4], the authors have studied the transient stability of a system by taking into account the operating conditions and disturbances in the power system. Transient stability of a (or any) power system mainly depends on the network parameters and operating conditions of the power system, fault type and location, and fault clearing time. A transient energy function method for transient stability analysis has been reviewed in [5]-[7]. These review works have been presented based on some mathematical models and elementary theory. Global controls of single-machine to an infinite bus have been proposed by studying the transient and voltage regulation in [8]. Here, a global controller has been designed to coordinate transient stabilizer and voltage regulator. The IPSA software has been used to study transient stability analysis of Gadong power station by considering a three-phase fault at a selected busbar [9], where the generator's performance has been studied by considering the fault.

In this paper, a new method has been presented for analyzing transient stability of a faulted two-machine system. This method uses a set of closed loop transfer function for each machine and derives the dominant root by taking Laplace transformation of the machine's nonlinear equation. Using this proposed method and, the CYME power system software, the transient stability of

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the two-machine faulted system has been studied during different fault clearing times with respect to generator's relative rotor angle and the bus voltage.

II. Proposed Method

Position The well-known classical swing equation [10], [11], which is related to synchronous generator rotor swing angle is given by

$$M\frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad (1)$$

where

$$M = \frac{2H}{\omega_s} \quad (2)$$

The electrical power output of a single machine connected to an infinite bus at any instant of time is,

$$P_e = P_{max}\sin\delta \quad (3)$$

The mechanical power input equal to the prefault electrical power output at an initial angle δ_0 is given as,

$$P_m = P_{max}\sin\delta_0 \quad (4)$$

The P_{max} will have two constant values P_1 and P_2 respectively for $0 \le t \le t_r$ [10]. The maximum value of the machine output power P_{max} can be represented as a two-valued step function as shown in Fig. 1.

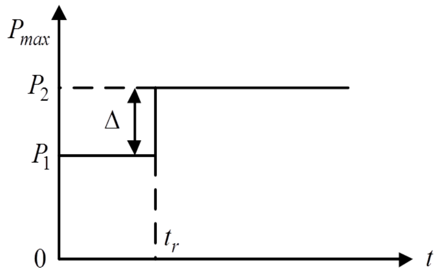

Figure 1. Maximum power output during and after faults.

From Fig. 1, an expression for P_{max} can be derived as

$$P_{max} = P_1 u(t) + \Delta u(t - t_r) \quad (5)$$

where,

$u(t)$ is the unit step function at $t = 0$,

$u(t - t_r)$ is the unit step function at $t = t_r$,

$$\Delta = P_2 - P_1$$

The rotor swing angle δ, which has been defined relative to a reference axis rotating at the synchronous angular velocity ω_s, can be related to the actual rotor speed ω as

$$\delta = (\omega - \omega_s)t = \omega' t \quad (6)$$

Differentiating equation (6) with respect to t provides,

$$\omega' = \frac{d\delta}{dt} = 2\pi f' \quad (7)$$

The ω' is a measure of the frequency (f') of rotor oscillation with respect to the synchronously rotating reference axis. Consider the following relation:

$$y = \sin\delta \quad (8)$$

Substituting equation (6) into equation (8), yields,

$$y = \sin\omega' t \quad (9)$$

Combining equations (3) and (8) gives,

$$P_e = P_{mxm} y \quad (10)$$

Substituting equations (5) and (9) into equation (10) provides,

$$P_e = P_1 \sin\omega' t\, u(t) + \Delta\sin\omega' t\, u(t - t_r) \quad (11)$$

Taking Laplace transform of equations (1), (6), (9) and (11), yields,

$$Ms^2\delta(s) = P_a(s) = P_m(s) - P_e(s) \quad (12)$$

$$\delta(s) = \frac{\omega'}{s^2} \quad (13)$$

$$Y(s) = \frac{\omega'}{s^2 + \omega'^2} \quad (14)$$

$$P_e(s) = \frac{P_1\omega'}{s^2 + \omega'^2} + \frac{\Delta e^{-t_r s}(\omega'\cos\omega' t_r + s\sin\omega' t_r)}{s^2 + \omega'^2} \quad (15)$$

Equation (12) provides the transfer function of the forward path block of Fig. 2 and it can be expressed as,

$$G(s) = \frac{\delta(s)}{P_a(s)} = \frac{1}{Ms^2} \quad (16)$$

Dividing equation (14) by equation (13), yields the transfer function of the first feedback block,

$$H_1(s) = \frac{Y(s)}{\delta(s)} = \frac{s^2}{s^2 + \omega'^2} \quad (17)$$

Dividing equation (15) by equation (14) gives the transfer function of the second feedback block as,

$$H_2(s) = \frac{P_e(s)}{Y(s)} = P_1 + \frac{\Delta e^{-t_r s}(\omega'\cos\omega' t_r + s\sin\omega' t_r)}{\omega'} \quad (18)$$

The feedback model along with different components of the swing equation is shown in Fig. 2. The closed loop transfer function of the feedback system corresponding to a synchronous machine is,

$$\frac{\delta(s)}{P_m(s)} = \frac{G(s)}{1 + G(s)H_1(s)H_2(s)} \quad (19)$$

Substituting the expressions for $G(s)$, $H_1(s)$, $H_2(s)$ into equation (19) gives,

$$\frac{\delta(s)}{P_m(s)} = \frac{\frac{1}{M}\left(s^2+\omega'^2\right)}{s^2\left(s^2+k_1+k_2e^{-t_r s}+k_3 s e^{-t_r s}\right)} \tag{20}$$

where the expressions of k_1, k_2 and k_3 [9] are,

$$k_1 = \omega'^2 + \frac{P_1}{M} \tag{21}$$

$$k_2 = \frac{1}{M}\Delta\omega'\cos\omega' t_r \tag{22}$$

$$k_3 = \frac{\Delta\sin\omega' t_r}{M\omega'} \tag{23}$$

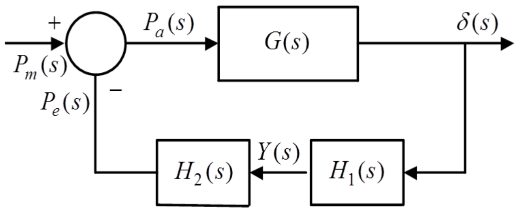

Figure 2. Swing equation into a feedback model.

The characteristic equation of the feedback system derived from the swing equation of a synchronous machine is the denominator of equation (20) equated to zero as shown below:

$$s^2+k_1+k_2e^{-t_r s}+k_3 s e^{-t_r s} = 0 \tag{24}$$

$$(s^2+k_1)-k_2e^{-t_r s}\left(-1-\frac{k_3 s}{k_2}\right)=0 \tag{25}$$

Equation (25) can be rewritten as follows:

$$f(s) = f_1(s) - f_2(s) = 0 \tag{26}$$

where,

$$f_1(s) = s^2 + k_1 \tag{27}$$

$$f_2(s) = k_2e^{-t_r s}\left(-1-\frac{k_3}{k_2}s\right) \tag{28}$$

From equation (26), it is evident that the root is the point of intersection of the functions $f_1(s)$ and $f_2(s)$. This point has been termed as the dominant root. Fig. 3 shows the sketches of the forms of the functions $f_1(s)$ and $f_2(s)$.

From Fig. 3 it can be seen that function $f_1(s)$ is greater than $f_2(s)$ i.e. $f(s) = f_1(s) - f_2(s)$ is positive for all values of s on the right of and up to the point $s = -{k_2}/{k_3}$. At $s = -{k_2}/{k_3}$, the function $f_2(s)$ is zero and beyond this point $f_2(s)$ starts increasing, so that $f_2(s)$ intersects $f_1(s)$ at a point having its real part equal to s_{root}. This point is the desired dominant root to be searched for. Beyond this point s_{root}, $f_2(s)$ is greater than $f_1(s)$ i.e., the function $f(s)$ is negative.

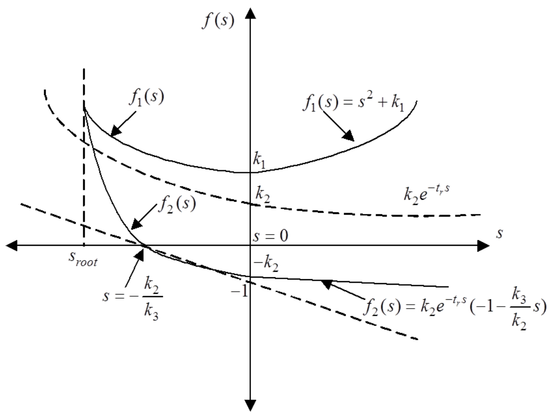

Figure 3. Sketches of functions $f_1(s)$ and $f_2(s)$.

Therefore, a search for the real axis bounds of the region containing the dominant root can be made starting from the point $s = -{k_2}/{k_3}$ and continued by increasing the absolute value of s until the function $f(s)$ becomes negative.

III. Results and Discussion

A two-machine power system with five lines, two transformers, and two loads, as shown in Fig. 4, has been studied. In this study, all data considered are in per unit on the same base as mentioned in Table I. A three-phase fault is assumed to occur at the sending end point P near bus 4 on the line 4-5, as shown in Fig. 4.

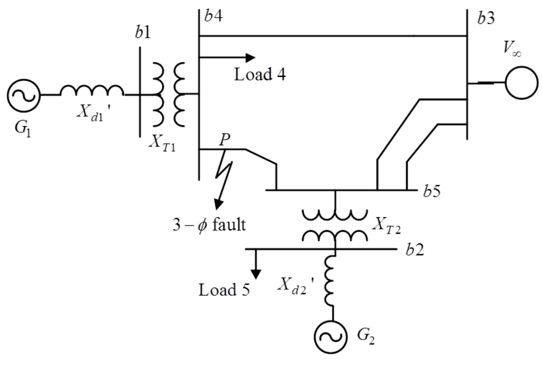

Figure 4. Single line diagram of a two-machine system.

The system has been studied by the proposed method and the CYME software, considering various fault clearing times of 0.05, 0.07, 0.10, 0.15 and 0.225 seconds, respectively. The CYME is a commercial power engineering application software, which can be used for Volt/VAR optimization, low voltage distribution modeling, advanced protective device coordination, voltage and transient stability analysis.

TABLE I. TWO MACHINE DATA IN PU ON 230 KV, 100 MVA BASE

Bus to bus	Series impedance	Shunt admittance
Tr1. 1-4	$0+j0.022$	7
Tr2. 2-5	$0+j0.040$	5
Line 3-4	$0.007+j0.040$	$0+j0.082$
Line 3-5 #1	$0.008+j0.047$	$0+j0.098$
Line 3-5 #2	$0.008+j0.047$	$0+j0.098$
Line 4-5	$0.018+j0.110$	$0+j0.226$
Load 4	$P+jQ=1+j0.44$	
Load 5	$P+jQ=0.50+j0.16$	
G1	400MVA, 20kV, $X_d{}'=0.067$, $H=11.2$ MJ/MVA	
G2	250MVA, 18kV, $X_d{}'=0.10$, $H=8.0$ MJ/MVA	

For the analysis, using the CYME software, the per unit value of the line, the actual values of the load, transformer and machine data are entered in the software window. Then the fault occurrence and fault clearing times (cycles) are inserted and, at the same fault clearing time instants line 4-5 is set to be disconnected in the software window. Subsequently, the load flow solution using the fast decoupled method is run. Finally, the transient stability analysis is carried out by monitoring generators' performance with respect to rotor's swing angle and bus voltage, for the simulation time of 2 seconds. The relative rotor angles for the two machines have been computed at an interval of $\Delta t = 0.01$ seconds and then plotted in $\delta - t$ plane with respect to the network, i.e., bus 3. These rotor angle curves for the fault clearing times of 0.05, 0.07, 0.10, 0.15 and 0.225 seconds are shown in Figs. 5 to 9, respectively.

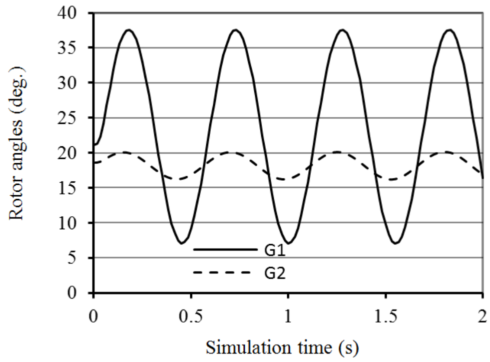

Figure 5. Variation of relative rotor angles for fault clearing times of 0.05 s.

From Figs. 5 to 8, it is observed that the relative rotor swing angles of both machines vary together with the simulation time, which represents that the generator 1 and generator 2 are stable for the first four fault clearing times. Also, during these first four fault clearing times, it is found that the magnitude of the relative rotor angles of generator 1 increases with an increase in the fault clearing times. On the other hand, for the fault clearing time of 0.225 seconds, as shown in Fig. 9, it is observed that the relative rotor angle of generator 1 increases sharply while the relative rotor angle of generator 2 remains constant instead of oscillating with the simulation time. These characteristics indicate that both generators are unstable for the fault clearing time of 0.225 seconds.

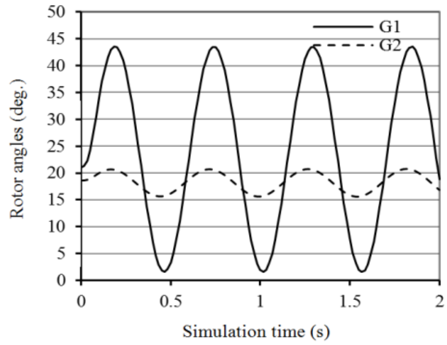

Figure 6. Variation of rotor angles for fault clearing times of 0.07 s.

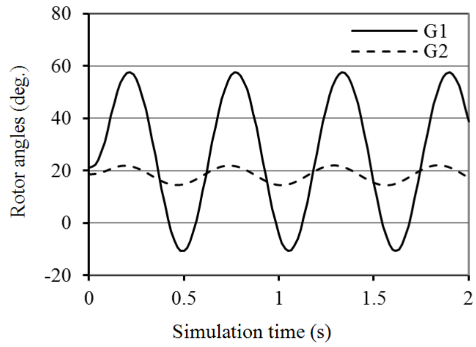

Figure 7. Variation of rotor angles for fault clearing times of 0.10 s.

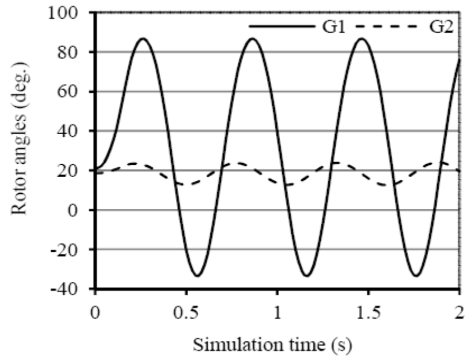

Figure 8. Variation of relative rotor angles for fault clearing times of 0.15 s.

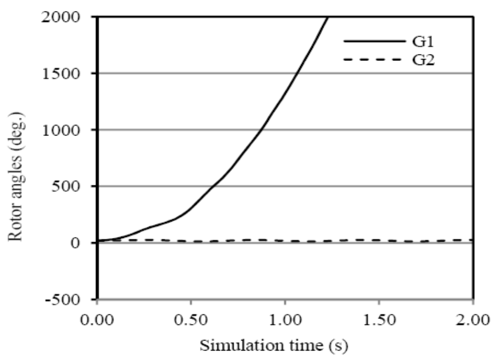

Figure 9. Variation of relative rotor angles for fault clearing time 0.225s.

The variation of bus voltages with respect to the fault clearing times of 0.05, 0.10, 0.15 and 0.225 seconds are shown in Figs. 10, 11, 12 and 13, respectively. From Figs. 10 to 12, it is observed that the bus 1 and bus 4 voltages are oscillating with respect to the simulation time after the fault clearing times, while the voltages of other buses remain constant. Whereas in Fig. 13, there is a trend of oscillation for all the bus voltages, which indicates that both generators of this system are unstable for the fault clearing time of 0.225 seconds. The results obtained by the proposed method by considering a rotor oscillation frequency of 0.25 Hz for both the machines have been shown in Table II, and again the unstable scenario has been observed for the fault clearing time of 0.225 seconds.

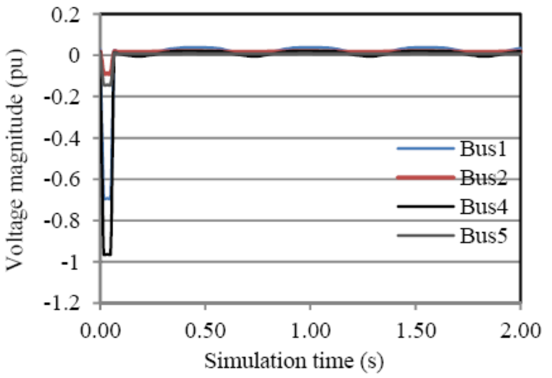

Figure 10. Variation of bus voltage for fault clearing times of 0.05 s.

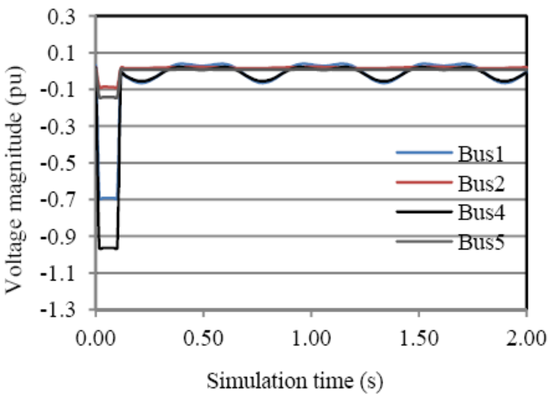

Figure 11. Variation of bus voltage for fault clearing times of 0.10 s.

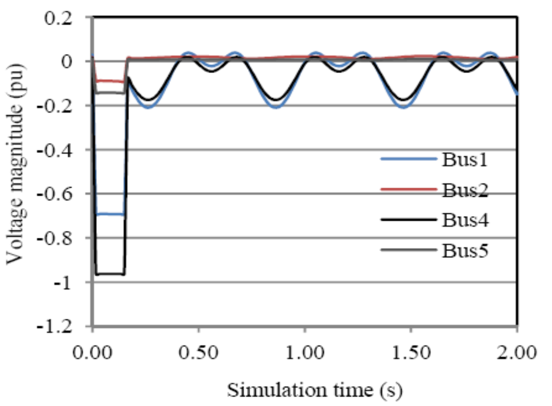

Figure 12. Variation of bus voltages for fault clearing time of 0.15 s.

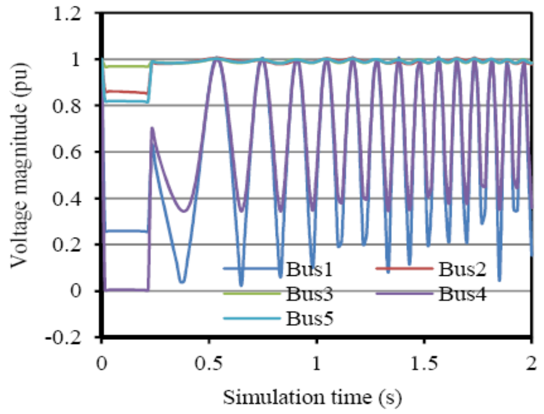

Figure 13. Variation of bus voltages for fault clearing time of 0.225 s.

TABLE II. LOCATION OF DOMINATING ROOTS FOR A OSCILATING FREQUENCY OF 0.25 HZ

Fault clearin g time (s)	Starting point for root search	Average number of search steps in increment of -10	Average lower limits of roots locations in s-plane	Remarks
0.05	-19.95	7	-69.45	Stable
0.07	-14.23	5	-39.23	Stable
0.10	-9.91	3	-21.42	Stable
0.15	-6.54	2	-11.04	Stable
0.225	-4.25	2	-6.75	unstable

By observing all the above-mentioned results it can be summarized that, at an operational stage, the transient stability analysis needs to be performed very fast as a part of the security control, so that appropriate decision and precautionary measures may be taken ahead of the occurrence of a fault.

IV. Conclusion

The proposed new method was used for analyzing transient stability analysis of a two-machine five bus power system network. The dominant root was searched in the s-plane for each generator starting from a real value which was same for both generators. The dominant root was found to be a function of the rotor oscillation frequency and the fault clearing time only. The minimum average distance of -11.04 was considered in s-plane for the two-machine system. An increment of -10 was found quite satisfactory for searching the dominant root. In the stable cases of the system, the average distance of the dominant roots was at or further left of the minimum average distance depending upon the fault clearing time. With higher fault clearing time, the average location shifted more towards right, resulting in decreased stability, so that, for unstable cases, the average distance was completely on the right of the minimum allowable value. The simulation results obtained using CYME power system software for the same fault clearing times were compared with the proposed method and were found in good agreement as mentioned in Table II and Fig. 9.

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