Reduction of PAPR of OFDM Symbols Using Modified Narahashi and Nojima Phasing Sequences

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Abstract—**This paper discusses the limitation of the phasing technique proposed by Narahashi and Nojima for reducing the PAPR of OFDM symbols. It is shown that although the phasing sequence proposed by Narahashi and Nojima works well for constant amplitude and constant phase angle signals, the technique does not work for inputs with varying phase angles, for example, QPSK. To improve this situation a modified phasing sequence is proposed and numerical results show a very good reduction in PAPR for input symbols with both constant and varying phase angles.**

Index Terms—**OFDM, peak-to-average power ratio, phasing schemes**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is gaining popularity in wireless communications due to its high data rate capability and robustness to hostile channel interference [1]. One major drawback of OFDM is its large peak-to-average power ratio (PAPR), which results in reducing the efficiency of the RF power amplifier used in the transmitter. Several techniques have been proposed in the literature, and an overview of various PAPR reduction techniques is available in [2].

Phasing methods have been investigated by many researchers [3]-[4] as a tool for reducing PAPR, wherein the initial phase of each tone is appropriately set to minimize the PAPR. While it has been shown in [3] that a PAPR of 2.60dB could be achieved by using a Newman phasing scheme [5], Narahashi and Nojima (NN) [6] have proposed an improved phasing method. The PAPR values achieved with their phasing scheme never exceeded 3.01dB and show improved PAPR reduction over Newman phasing for tone numbers 3, 4, 6, 7, 9, 12, 14, 17, 22, 24, and 29 [6]. However, the phasing scheme proposed in [6] gives the stated PAPR values only if the complex tones have constant amplitude and zero or constant phase shifts. As a result the phasing method cannot be used with applications involving digital modulation schemes such as phase shift keying (PSK) or quadrature amplitude modulation (QAM). For example, with quadrature phase shift keying (QPSK) modulation of random input signals are mapped to produce an output that could have one of four different phase angles. The derivation of the phasing scheme in [6] does not consider this inherent phase shift associated with each of the QPSK symbols, and therefore, the phasing scheme does not produce a reduced PAPR for OFDM systems that use digital baseband modulation.

This paper proposes a modification to the phasing scheme proposed by NN [6]. The proposed modification is derived from the instantaneous envelope power of an OFDM signal, and with an assumption that the baseband modulation is performed using QPSK. However, the formulation obtained could be used for any PSK methods.

The Newman phasing scheme [5] also suffers similar limitations in that it cannot reduce the PAPR if the input symbols are random. The proposed modification can also be applied to the Newman phasing scheme and the reduction of PAPR is equally good for both constant amplitude or varying amplitude input signals.

One of the drawbacks of the NN, the Newman and the proposed phasing schemes is that the receiver must have a-priori knowledge of the phase shift being given to each symbol, or the information must be transmitted as side information. This will increase the transmission overhead.

This paper is structured as follows. The proposed modification to the NN phasing scheme is derived in Section 2. Section 3 provides the results of numeric calculations and simulation, where a comparison of PAPR obtained using the proposed method is compared with that obtained using NN phasing. Section 4 is a conclusions section and summarises the main results from the letter.

II. Derivation of Phasing Scheme

The instantaneous amplitude of a baseband OFDM signal can be written as

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$$x(t)=\sum_{n=0}^{N-1}\widetilde{X}_n \exp j(2\pi nt/T) \qquad (1)$$

where $\widetilde{X}_n = X_n \exp j(\Phi_n)$ is the complex QPSK symbol, where Φ_n is the phase angle of the QPSK symbol, n represents the number of the subcarrier and T is the symbol duration. The phasing scheme derived in [6] would reduce PAPR only if the phase angle, Φ_n, is a constant for all symbols, however, QPSK mapping of random input symbols results in varying values for Φ_n. Therefore, it is important to consider the phase angle of QPSK symbols in the derivation of the proposed phasing scheme.

We can rewrite (1), as

$$x(t)=\sum_{n=0}^{N-1}X_n \exp j(\Phi_n + 2\pi nt/T) \qquad (2)$$

In an attempt to reduce the PAPR of OFDM signal, each QPSK symbol is given an initial phase shift. The OFDM signal as a result of the phase-shifted QPSK symbols can then be represented as follows:

$$x_1(t)=\sum_{n=0}^{N-1}X_n \exp j(\Phi_n + \theta_n + 2\pi nt/T)$$

where $\theta_n = [\theta_0, \theta_1, \ldots, \theta_{N-1}]$ is the initial phase shift applied to each QPSK symbol. The magnitude of QPSK symbols is unity and, therefore, the above equation can be written as

$$x_1(t)=\sum_{n=0}^{N-1}\exp j(\Phi_n + \theta_n + 2\pi nt/T) \qquad (3)$$

The instantaneous envelope power is given by,

$$P_1(t)=|x_1(t)|^2=\left\{\sum_{n=0}^{N-1}\exp j(\Phi_n+\theta_n+2\pi nt/T)\right\} \cdot \left\{\sum_{n=0}^{N-1}\exp -j(\Phi_n+\theta_n+2\pi nt/T)\right\}$$

$$= N + 2\sum_{n=0}^{N-2}\sum_{m=n+1}^{N-1}\cos\Psi \qquad (4)$$

where

$$\Psi = (\Phi_n - \Phi_m + \theta_n - \theta_m + 2\pi(n-m)t/T)$$

The average envelope power is given by,

$$P_{avg}(t) = E\left\{|x_1(t)|^2\right\}$$

where $E\{.\}$ is the expectation value. Using (1), we get an expression for average power as follows,

$$E\left\{|x_1(t)|^2\right\} = E\left\{\sum_{n=0}^{N-1}|X_n|^2 + 2\sum_{n=0}^{N-2}\sum_{m=n+1}^{N-1}X_n X_m \cos\Psi\right\}$$

For orthogonal signals, as the peak of one symbol falls at the null of other symbol, the second term in the above equation reduces to zero. Therefore, for QPSK, the average power of the OFDM signal under consideration is therefore given by

$$E\left\{|x_1(t)|^2\right\} = E\left\{\sum_{n=0}^{N-1}|X_n|^2\right\} = E\left\{\sum_{n=0}^{N-1}1\right\} = N \qquad (5)$$

Peak-to-Average Power Ratio (PAPR)

PAPR is defined as the ratio of the maximum (peak) of the instantaneous envelope power to the average power. Thus,

$$PAPR = \frac{\max\left\{|x_1(t)|^2\right\}}{E\left\{|x_1(t)|^2\right\}} \qquad (6)$$

Substituting (4) and (5) in (6),

$$PAPR = \frac{1}{N}\max\left\{N + 2\sum_{n=0}^{N-2}\sum_{m=n+1}^{N-1}\cos\Psi\right\}$$

$$= \max\left\{1 + \frac{2}{N}P(t)\right\} \qquad (7)$$

where,

$$P(t) = \sum_{n=0}^{N-2}\sum_{m=n+1}^{N-1}\cos\Psi \qquad (8)$$

Equation (8) can be written as

$$\begin{aligned} P(t) = &\sum_{k=0}^{N-2}\cos\{2\pi t/T + \Phi_{k+1} - \Phi_k + \theta_{k+1} - \theta_k\} \\ &+ \sum_{k=0}^{N-3}\cos\{2\pi 2t/T + \Phi_{k+2} - \Phi_k + \theta_{k+2} - \theta_k\} \\ &+ \sum_{k=0}^{N-4}\cos\{2\pi 3t/T + \Phi_{k+3} - \Phi_k + \theta_{k+3} - \theta_k\} \\ &\vdots \\ &+ \cos\{2\pi(N-1)t/T + \Phi_{N-1} - \Phi_0 + \theta_{N-1} - \theta_0\} \end{aligned} \qquad (9)$$

P(t) obtained in (9) represents the fluctuations from average power of the OFDM transmission. The objective is to find the initial phase angles that minimize *P(t)*. It is difficult to obtain solutions considering all terms in the right-hand side of (9). Based on the solution proposed in [6], which considers only the first summation term of *P(t)*, a solution to (9) can be formulated as shown below in (10).

$$\begin{aligned} &[(\Phi_{k+1} - \Phi_k) - (\Phi_k - \Phi_{k-1})] \\ &+ [(\theta_{k+1} - \theta_k) - (\theta_k - \theta_{k-1})] = \frac{2\pi}{N-1} \end{aligned} \qquad (10)$$

Rearranging the terms in (10), gives

$$\begin{aligned} &[\{(\Phi_{k+1} + \theta_{k+1}) - (\Phi_k + \theta_k)\} \\ &- \{(\Phi_k + \theta_k) - (\Phi_{k-1} + \theta_{k-1})\}] = \frac{2\pi}{N-1} \end{aligned}$$

Replacing $(\Phi + \theta)$ with φ, we get,

$$\left[(\varphi_{k+1}-\varphi_k)-(\varphi_k-\varphi_{k-1})\right]=\frac{2\pi}{N-1} \tag{11}$$

Using the solution to φ_k proposed in [6], we get (assuming $\varphi_1=\varphi_2=0^\circ$),

$$\varphi_k=\frac{\pi(k-1)(k-2)}{N-1}$$

and therefore,

$$\theta_k=\frac{\pi(k-1)(k-2)}{N-1}-\Phi_k \tag{12}$$

Equation (12) gives an algebraic expression for the initial phase shift to be added to QPSK symbols.

TABLE I. PAPR COMPARISON

No. of subcarrier	PAPR (dB)		
	Modified	NN phasing	Theoretical maximum (N logN)
2.00	3.01	3.01	3.01
3.00	2.22	4.77	4.77
4.00	2.58	5.17	6.02
5.00	2.92	3.44	6.99
6.00	2.57	4.91	7.78
7.00	2.70	5.73	8.45
8.00	2.89	4.61	9.03
9.00	2.48	5.04	9.54
10.00	2.76	6.91	10.00
12.00	2.62	6.06	10.79
14.00	2.71	7.06	11.46
16.00	2.80	6.47	12.04
18.00	2.81	4.63	12.55
20.00	2.75	5.48	13.01
22.00	2.68	4.75	13.42
24.00	2.68	6.05	13.80
26.00	2.74	6.69	14.15
28.00	2.74	8.18	14.47
30.00	2.76	6.21	14.77
32.00	2.75	5.55	15.05
64.00	2.69	7.27	18.06
100.00	2.65	8.07	20.00
128.00	2.64	7.34	21.07
200.00	2.61	9.89	23.01
256.00	2.62	7.75	24.08
300.00	2.61	7.35	24.77

III. NUMERICAL AND SIMULATION RESULTS

The PAPR due to the proposed phasing scheme is numerically computed and compared with the PAPR due to the NN phasing method and also with the theoretical maximum PAPR when no phasing method is used. For computation of PAPR using NN phasing, the information source used is a random binary sequence, which is mapped using QPSK modulation. This ensures that the input symbols do not have a constant initial phase shift and helps to prove that the proposed modified phasing scheme is better that the NN phasing scheme, especially when any OFDM system deals with random input signals.

A. *Modified vs. NN Phasing Schemes*

Table I gives the values of PAPR for the proposed modified phasing scheme as a function of the number of subcarriers and compares this with the PAPR for NN phasing and the maximum theoretical PAPR value when no phasing scheme is employed.

It can be seen that though NN phasing can produce a PAPR as low as 2.65dB [6] when the input symbols are of constant amplitude and constant inherent phase shift, it does not produce low PAPR when the input signal is random. However, the modified phasing scheme suggested in this paper produces a PAPR as low as about 2.61dB. As the number of subcarriers increases beyond 200, the PAPR converges to around 2.61dB.

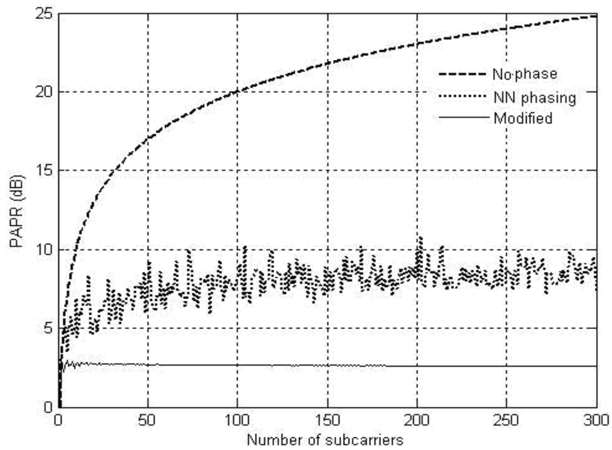


Figure 1. Simulated PAPR vs. number of subcarrier for modified and NN phasing schemes.

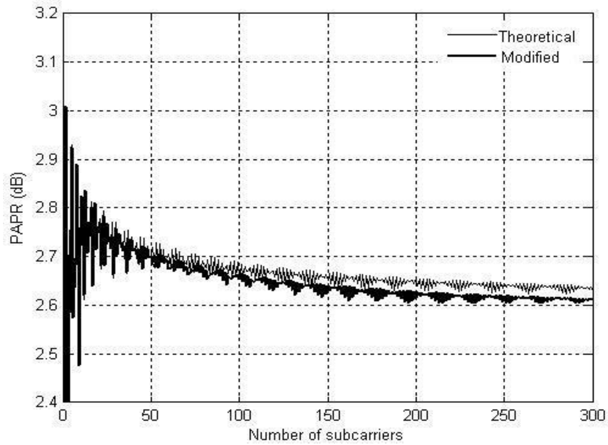


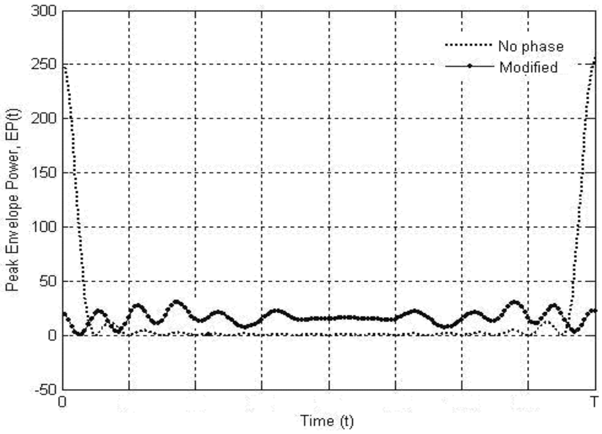
Figure 2. Theoretical and simulated PAPR vs. number of subcarriers

The numeric results obtained have been verified to match with the simulation results. An OFDM system with various phasing schemes has been simulated using Matlab, and the results are shown in Fig. 1. It can be seen that the modified phasing scheme produces a much reduced PAPR level when compared to the NN phasing method.

Fig. 2 compares the PAPR values due to the modified phasing schemes that are obtained theoretically using (8) and through simulation. It is also noted that the PAPR from simulation closely matches with the theoretical value, thus validating the simulation model.

B. Peak Envelope Power

Fig. 3 provides a plot of peak envelope power of a simulated OFDM system containing 16 subcarriers with and without the modified phasing scheme employed. This plot clearly illustrates the ability of the phasing scheme in containing the peak envelope power to within 3.01dB of the average power.



Peak envelope power vs. time (16 subcarriers)

IV. Conclusion

A modified phasing scheme is introduced that overcomes the limitation of NN phasing to reduce the PAPR of OFDM systems. The proposed modification is easy to implement and results in a significant reduction in PAPR for systems using either constant amplitude input signals or random input signals.

The proposed phasing scheme has been validated through simulations and following results are obtained. The PAPR is contained within 3.01dB for OFDM systems with up to 300 subcarriers. The PAPR converges to about 2.61dB as the number of subcarriers approaches and increases above 300.

A common drawback of such phasing schemes is that the receiver must have prior knowledge of the phase shift being given to each symbol, and hence the transmission overhead is increased. Investigations on reducing the overhead or enabling reception without a prior knowledge of the phase shift require to be further investigated.

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