An Efficient Method of Computation for Jammer to Radar Signal Ratio in Monopulse Receivers with Higher Order Loop Harmonics

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Abstract—Monopulse radar receivers are the most advanced tracking receivers which often employ phase locked loop (PLL) for coherent detection of the received echo signal to become jam resistance and clutter rejection. In this paper, the loop error in the monopulse receiver is characterized analytically taking several higher order harmonics of input signal phase into account. The effects of these harmonics in breaking the frequency lock in the receiver are studied. The phasor diagram is used as an index to determine input signal phase for the given voltage controlled oscillator (VCO) phase and loop phase error at different values of frequency separation between radar echo signal and interference signal. This allows in computing additional phase error and jammer to radar echo signal amplitude (J/S) ratio required for jamming the receiver. The result shows that for the given phase error, the receiver requires large J/S ratio for break-lock when several higher order harmonics are taken into account. It is also shown that the error results due to higher order harmonics is positive and linear. Thus, the additional J/S ratio required for break-lock can be predicted from the error when fundamental harmonics is considered alone. This reduces the complex computation of J/S ratio required for break-lock when higher order harmonics are considered in the loop analysis. The analysis of the loop taking higher order harmonics into account is carried out for typical loop damping ratio of 0.707 and 1.0.

Index Terms—monopulse receiver, loop phase error, phasor diagram, radar echo, tracking receivers

I. INTRODUCTION

There are several types of receivers which are in vogue for missile seeker applications. Missile borne monopulse radar receivers employing phase locked loop (PLL) are mainly used to determine the positional resolution of single target based upon Doppler frequency drift relative to the PLL bandwidth and relative amplitude of the radar echo and interference signal [1]. These monopulse receivers track the target in frequency, angle and range domains. Particularly, in the frequency domain, so long as the missile receiver tracks the target, the missile maintains its course and the receiver locks onto the received radar echo frequency. The break-lock in the PLL occurs when continuous wave (CW) sinusoidal signal in the form of deliberate noise or interference signal is injected into the radar receiver along with the desired radar echo signal [2]. This leads to instability in the receiver in tracking the target and thus the missile lose the frequency track of the target and is miss guided. Several studies have been carried out on jamming the missile radar receivers earlier [3]-[5]. It is observed that carrier synchronization system performance is degraded by both additive noise and interference signal due to crowding of useful and unwanted frequency spectrum [6]. The critical interference to signal power ratio beyond which the loop loses lock has been studied in [7], [8]. In a specific case study, the received signal is analyzed by expanding its time varying phase in Fourier series using Describing function method by considering fundamental harmonic component only [9]. In this paper, several higher order harmonics of the loop input signal phase have been considered and their effects on break-lock conditions in the receiver have been studied. The phasor diagram that relates the input signal phase (σ), output phase of voltage controlled oscillator (η) and loop phase error (ε) is plotted for different values of frequency separation (Δω) between the radar echo and the interference signal taking several higher order harmonics into account. The frequency separation between the two signals is expressed in terms of loop bandwidth. From the phasor diagram, the jammer to radar echo signal amplitude (J/S) ratio is computed for a given loop phase error when several higher order harmonics are taken into account. This allows in predicting additional J/S ratio required for break-lock in the receiver when the fundamental harmonics is considered alone. Hence, it eliminates the complex computation of loop phase error and additional J/S ratio when higher order harmonics are included in the analysis. It is shown that for a given loop
phase error, the receiver requires large J/S ratio to break the frequency lock. It is also seen that the loop phase error depends upon frequency separation between the radar echo and interference signal. The phase error in the loop is more when the frequency separation between the two signals is large.

II. MONOPULSE TRACKING PRINCIPLE

The mono pulse radar receiver system is shown in Fig. 1.

As shown in the Fig. 1, the monopulse radar receiver employs two overlapping antenna patterns to obtain the angular error in one coordinate. The two overlapping antenna beams are generated with a single reflector or with a lens antenna illuminated by two adjacent feeds. The two adjacent antenna feeds are connected to the two arms of a hybrid junction. The sum and difference signals appear at the two arms of the hybrid. The sum pattern is used for transmission, while both the sum and difference patterns are used on reception. On reception, the outputs of the sum and difference arms are each heterodyned to an intermediate frequency and amplified as, in any superhetrodyne receiver. The transmitter is connected to the sum arm from which range information of the target is extracted. The signal received with the difference arm from which range information of the target is extracted. The signal received with the difference arm from which range information of the target is extracted.

III. MATHEMATICAL MODELING OF LOOP

Let’s consider the basic phase locked loop as shown in Fig. 2.

Let, the radar echo signal at the loop input is given by:

\[ A_1 = X_1 \sin(\omega_{if} t) \]  \hspace{1cm} (1)

and the interference signal is

\[ A_2 = X_2 \sin(\omega_{if} t + \Delta \omega t) \]  \hspace{1cm} (2)

where, \( X_1 \) and \( X_2 \) are the amplitudes of desired radar echo signal and interfering signal respectively, \( \omega_{if} \) is the frequency of radar echo and \( \Delta \omega \) is the frequency separation between the signals.

The input signal to the loop is given by

\[ Y(t) = X_1 \sin(\omega_{if} t + \Delta \omega t) + X_2 \sin(\omega_{if} t + \Delta \omega t) \]  \hspace{1cm} (3)

\[ = X(t) \sin[\omega_{if} t + \theta(t)] \]

where, \( X(t) \) is the time varying amplitude of the loop input signal which is given by

\[ X(t) = X_1(t) \sqrt{1 + \left( \frac{X_2}{X_1} \right)^2 + 2 \left( \frac{X_2}{X_1} \right) \cos \Delta \omega t} \]  \hspace{1cm} (4)

and \( \theta(t) \) is time varying phase of loop input signal and is given by

\[ \theta(t) = \tan^{-1} \left( \frac{\frac{X_2}{X_1} \sin \Delta \omega t}{1 + \frac{X_2}{X_1} \cos \Delta \omega t} \right) \]  \hspace{1cm} (5)

Now, expanding the time varying phase of input signal in Fourier series [9]:

\[ \theta(t) = R \sin \Delta \omega t - \frac{R^2}{2} \sin 2 \Delta \omega t + \frac{R^3}{3} \sin 3 \Delta \omega t + \ldots \ldots R < 1 \]

\[ \Delta \omega t - (R^2) \sin \Delta \omega t + \frac{R^2}{2} \sin 2 \Delta \omega t + \frac{R^3}{3} \sin 3 \Delta \omega t + \ldots \ldots R > 1 \]  \hspace{1cm} (6)

where, \( R \) is the jammer to radar signal amplitude ratio (J/S = \( X_2/X_1 \)). With reference to Fig. 2, the loop phase error \( \varepsilon(t) \) which is the difference between loop input phase \( \sigma(t) \) and VCO output phase \( \eta(t) \) can be written as:

\[ \varepsilon(t) = \sigma(t) - \eta(t) \]  \hspace{1cm} (7)

So, the input signal phase of the loop is given by

\[ \sigma(t) = \varepsilon(t) + \eta(t) \]  \hspace{1cm} (8)

and the phasor equation for the above equation (7) can be written as:

\[ \sigma = \varepsilon + \eta \]  \hspace{1cm} (9)

where \( \sigma \), \( \varepsilon \) and \( \eta \) are the phasor of \( \sigma(t) \), \( \varepsilon(t) \) and \( \eta(t) \) respectively.

IV. PHASE ERROR AND J/S RATIO COMPUTATION

Let’s assume that the interference signal is initially smaller than radar echo signal (\( R<1 \)) and the PLL tracks the radar echo signal frequency. The amplitude of interference signal is then increased to a value greater
than radar echo signal. For R>1, the mathematical analysis shows that, the phase detector output is written as:

$$
\epsilon(t) = \epsilon \sin \Delta \omega t + \Delta \omega t + \gamma - \phi = 180^\circ
$$  \hspace{1cm} (10)

where, \( \gamma \) is the phase of \( \sigma(t) \) and \( \phi \) is dc phase of VCO. So, the VCO output which is the filtered version of \( \epsilon(t) \) is given by

$$
\eta(t) = \sin(\epsilon \sin \Delta \omega t + \Delta \omega t + \gamma - \phi = 180^\circ)
$$  \hspace{1cm} (11)

Expanding \( \eta(t) \) in terms of fundamental harmonics, second and higher order harmonics, it is given by equations (12), (13) and (14) respectively as:

$$
\eta(t) = -|J_0 - J_2| \sin \Delta \omega t
$$  \hspace{1cm} (12)

$$
\eta(t) = -[J_0 - J_2] \sin \Delta \omega t - J_1 \sin 2\Delta \omega t
$$  \hspace{1cm} (13)

$$
\eta(t) = -[J_0 - J_2] \sin \Delta \omega t - J_1 \sin 2\Delta \omega t - J_3 \sin 3\Delta \omega t + ...
$$  \hspace{1cm} (14)

So, the input phasor equation (from equation (9)) for fundamental harmonics, second and higher order harmonics are obtained and are given in equation (15), (16) and (17) respectively as:

$$
\sigma = \epsilon - [J_0 - J_2]G(j\omega)
$$  \hspace{1cm} (15)

$$
\sigma = \epsilon - [J_0 - J_2]G(j\omega) - J_1G(j2\omega)
$$  \hspace{1cm} (16)

$$
\sigma = \epsilon - [J_0 - J_2]G(j\omega) - J_1G(j2\omega) - J_2 \sin 3\Delta \omega t G(j3\omega) + ...
$$  \hspace{1cm} (17)

where, \( J_0, J_1 \) and \( J_2 \) is zero order, first and second order Bessel function respectively and \( G(j\omega) \) is the loop transfer function and is given by:

$$
G(j\Delta \omega) = -\left[ -\frac{1}{(2\zeta)^2} \left( \frac{B}{\Delta \omega} \right)^2 - j\left( \frac{B}{\Delta \omega} \right) \right]
$$  \hspace{1cm} (18)

where, \( \zeta \) is loop damping ratio and \( B \) is loop bandwidth. So, from these phasor equations, the input signal phase is determined for a given loop phase error and then J/S ratio is obtained. From the equation (15), (16) and (17), the phasor diagram for \( \zeta = 0.707 \) & 1.0 and \( \Delta \omega = 1.4B, 2B \) and \( 5B \) are shown in Fig. 3 and Fig. 4.

In the Fig. 3(a) and Fig. 3(b), the phasor diagram for loop input signal phasor is drawn in the complex plane with fundamental, second and higher order harmonics. From these phasor diagrams, the J/S ratio required for break-lock is determined for the given loop phase error as the input signal phase is expressed in terms of J/S ratio (equation (5)). The variation in phase error as a function of J/S ratio for \( \zeta = 0.707 \) & 1.0 and \( \Delta \omega = 1.4B, 2B \) and \( 5B \) is shown in Fig. 4 and Fig. 5.
It is clear from the Fig. 4 and Fig. 5 that the phase error of the loop depends upon the frequency separation between the radar echo and interference signal. It is seen from these above figures that the phase error is typically 0.1, 0.4 and 0.8 when $\Delta \omega = 1.4B$, $2B$ and $5B$ respectively. Thus, it is verified that when the frequency separation between the two signals is large, the phase error in the loop is more. It is also seen from fig. 4 and fig. 5 that for a given phase error, the J/S ratio required for break-lock in the receiver with higher order harmonics is large as compared to fundamental harmonics. The additional J/S ratio is computed for different values of loop error and is shown in Fig. 6(a) and Fig. 6(b).

It is clear from the Fig. 6(a) and Fig. 6(b) that the change in J/S ratio with loop phase error is positive and linear regardless of frequency separation between the radar echo and interference signal. Thus, it allows in predicting additional J/S ratio required for break-lock in the receiver from the results when fundamental harmonics is considered alone. Hence, it can be concluded that higher order harmonics of the input signal should be taken into account when the receiver jamming is a serious issue and should not be neglected.

V. CONCLUSION

In this paper, the loop phase error and J/S ratio required for break-lock have been computed and reported with several higher order harmonics of the input signal phase. It is verified that for the given loop phase error, the J/S ratio required for breaking the frequency lock in the receiver is large when several higher order harmonics are considered. It is verified that the change in additional J/S ratio with loop phase error is positive and linear regardless of frequency separation between the radar echo and interference signal. Thus, it is recommended that large frequency separation between these two signals should be maintained for effective jamming of the radar receiver. Hence, it can be concluded that higher order harmonics of the input signal should be taken into account when the receiver jamming is a serious issue and should not be neglected.

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