Compromise between Guarg Interval and Orthogonality in OFDM Transmission

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***Abstract*—Transmission of higher rates on frequency selective channels is nowadays a challenge. The multi-carrier modulations can made a good way to achieve the desired rates on this type of channels. Based on the simultaneous transmission of user data on a plurality of subcarriers modulated narrowband multiplexing, they allow, by increasing for each carrier symbol duration beyond the channel impulse response, to minimize IES. The frequency selective channel can thus be considered as non- selective for each of the sub -carriers. However, a major drawback was the spectra of different carriers that are necessarily disjoint, what caused decreased spectral efficiency. To reduce the frequency band occupied, the spectrum of sub-carriers must cover each other while trying to reduce or eliminate the interference between subcarriers. To compensate for this loss, optimization of the occupied band is introduced by a partial recovery of the spectra of different subcarriers. Conditions of orthogonally between the subcarriers has been demonstrated leading to possible recovery of their respective spectra.**

***Index Terms*—ISI, ICI, FDM, OFDM, DFT, IFFI, FFT, CP, CS, ZF**

I. Introduction

In Multicarrier systems, the minimum spacing between carriers strongly dependent on the choice of the waveform, is equal to $1/T_s$ typically in the case of the door function. Using this basic function defines a frequency multiplex of orthogonal subcarriers, most often referred to, by the abbreviation OFDM. However, these criteria alone do not maintain the orthogonality between sub-carriers in the spread of a multipath channel where the IES appears in reception. To eliminate this interference without increasing the symbol duration T_s or equivalently the number of subcarriers, one solution is to deliberately sacrifice a portion of the emitted energy to precede each symbol with a guard interval Δ of duration T_Δ selected higher or equal to the spreading of the channel impulse response. In this way, the useful part of T_s duration of each OFDM symbol is not affected by IES reception since it is absorbed by the guard interval [1]. Note that as the interval between the subcarriers is always equal to $1/T_s$ while the duration of an OFDM symbol is now equal to T_s+T_Δ, the orthogonality between subcarriers is lost by inserting the guard interval. It will be restored after deleting Δ in reception. This insertion causes a loss of power equal to $10\log_{10}\left(\frac{T_s}{T_s+T_\Delta}\right)$ and a loss in spectral efficiency equal to à$\frac{T_s}{T_s+T_\Delta}$. For not using the guard interval while limiting the IES, various functions different from the door function are studied ex. the IOTA function. These functions must first be well localized in time and frequency, and secondly they must be orthogonal [2]. In this paper we discuss firstly the OFDM principle, the concept of orthogonality, then we describe the OFDM system and focus on the guard interval added to the system in order to fight inter symbol interference. As a conclusion of this work, results show the repercussion of the guard interval on OFDM system, in orthogonality environment.

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II. Principle of OFDM

In the case of propagation over a multipath channel, several replicas of the transmitted wave are received with different delays and amplitudes. It result then the inter-symbol interference (ISI). Conventional modulation techniques to transmit over such channels are very sensitive to this type of interference that are also particularly important when the symbol duration is small compared to the delay spread of the channel. There is therefore a tradeoff between throughput related to the symbol duration and reliability of the connection related to interference between symbols. The multi-carrier modulations provide an interesting optimization of the compromise [1]. The principle of multi -carrier modulation is based on the parallelism of the frequency information to be transmitted. Thus, the N data previously transmitted consecutively at a high rate of 1/ T_d will be transmitted simultaneously on N basic frequency subchannels or subcarriers modulated low flow $1/T_s$. Thus, each of the N data are transmitted by a symbol duration T_s instead of $T_d = \frac{T_s}{N}$. The symbol duration is multiplied by a factor N, which good design should minimize the ISI while keeping the flow of the original single-carrier modulation. In the time domain, the received signal is decomposed into symbols of duration T_s. In the frequency domain, the signal distortion introduced by the channel will be limited because each sub-band is narrow enough to consider the channel locally flat. Multicarrier modulations take benefit from a reduction of the complexity of the equalization stage in reception.

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III. Concept of Orthogonality

Minimizing ISI unfortunately is accompanied by a new term interference, the interference between carriers ICI. It results from the difficulty to separate the information transmitted simultaneously on different subcarriers. Recommended solution for the first FDM systems was to increase the spacing between the bands occupied by each one. This solution is however not optimal in terms of spectral efficiency and often leads to an occupation of a band twice as large as in the case of a single-carrier system. It is possible to maintain a high spectral efficiency by forming a frequency multiplex so that the spectra of the subcarriers overlap ensuring that it forms a base of orthogonal functions. The orthogonality constraints are defined in a time and frequency point of view [3].

Consider an assembly of fk frequencies as:

$$f_k = f_0 + k\Delta_f \qquad \forall\, k\ \in [0, \dots, N-1] \tag{1}$$

where f_0 is the original carrier frequency, Δ_f the difference between two consecutive subcarriers and the number N of subcarriers. A basic $\Psi_{j,k}(t)$ of elementary signals is defined by:

$$\psi_{j,k}(t) = g(t - jT_s)e^{2\pi i f_k t} \tag{2}$$

where $g(t)$ is any function defined on $[0,\ Ts]$ function called formatting function. These elementary signals form an orthogonal basis if the scalar product of two elementary signals is equal to:

$$\prec \psi_{j,k}(t), \psi_{j',k'}(t) \succ = \int_{-\infty}^{+\infty} \psi_{j,k}(t)\psi_{j',k'}{}^{*}(t)\, dt \qquad = E_\psi . \delta_{j,j'} . \delta_{k,k'} \tag{3}$$

E_Ψ is the energy function Ψ and $\delta_{l,m}$ is the Kronecker symbol:

$$\delta_{l,m} = \begin{cases} 1 & si\ l = m \\ 0 & si\ l \neq m \end{cases} \tag{4}$$

Depending on the choice of g(t) and Δ_f, the result of the scalar product of the above equation leads to an orthogonality of functions $\Psi_{j,k}(t)$ in time (j index) and/or frequency (k index).

IV. Temporal Orthogonality

It results in constraints on the choice of the function of formatting g(t). Works of R. W. Chang [3] have demonstrated that they result in conditions on the modulus and the argument of g(t). A detailed list with their advantages, disadvantages and applications is given by [4]. Among the many possibilities, the door function has proved to be the most frequently used for its simplicity of implementation. It consists of a rectangular windowing of OFDM symbols:

$$g(t) = \begin{cases} 1 & for\quad 0 \leq t \leq Ts \\ 0 & \qquad\qquad outsides \end{cases} \tag{5}$$

V. Frequency Orthogonality

As can be seen before, the choice of Δ_f is important for optimal recovery of subcarriers. The minimum distance between two consecutive sub-carriers strongly depends on the choice of the function. The previous equation defining the orthogonality basic functions must absolutely be checked whatever the two consecutive subcarriers:

$$\prec \psi_{j,k}(t), \psi_{j,k+1}(t) \succ = \int_{-\infty}^{+\infty} \psi_{j,k}(t)\psi_{j,k+1}{}^{*}(t)\, dt \qquad = 0 \qquad \forall k \tag{6}$$

So for door function:

$$\prec \psi_{j,k}(t), \psi_{j,k+1}(t) \succ = \int_{-\infty}^{+\infty} \Pi(t - jTs)e^{2\pi i f_k t}\Pi(t - jTs)e^{-2\pi i f_{k+1} t}\, dt \ = \ 0 \qquad \forall k \tag{7}$$

$$\prec \psi_{j,k}(t), \psi_{j,k+1}(t) \succ = \frac{e^{-2\pi i \Delta f Ts}\sin(\pi\Delta f Ts)}{\pi\Delta f} = 0 \tag{8}$$

This last equality holds if:

$$\pi\Delta f Ts = p\pi \tag{9}$$

$$\Delta f = \frac{p}{Ts} \qquad p \in \mathbb{Z}^* \tag{10}$$

This equation can give a perfect orthogonality between different subcarriers as shown in Fig. 1. However, for p≠1, the recovery of subcarriers is not optimal. In practice, we seek to ensure that the bandwidth occupied by the signal is as low as possible. Therefore, the difference Δ_f between two consecutive sub-carriers shall be as low as possible, so:

$$\Delta f = \frac{1}{Ts} \tag{11}$$

This calculation for the door function, can be repeated for all other functions.

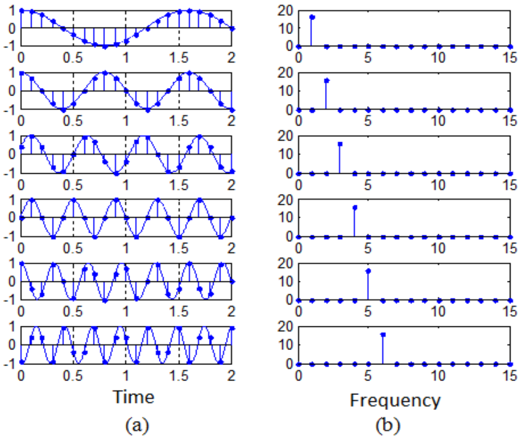


Figure 1. Presentation of principle of frequency orthogonality (a): time domain (b): frequency domain

VI. OFDM Systme Description

As we have seen, the OFDM signal consists of N subcarriers frequency f_k=f_0+$k\Delta_f$ used for parallel transmission of N symbols. These symbols are denoted xk complex elements taking values in a finite alphabet corresponding to a given modulation such as phase modulation. In the case where the function of forming is the door function the expression of the OFDM signal generated in the interval [0, Ts] is given by:

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=o}^{N-1} \Re\{x_k \Pi(t) e^{2\pi i f_k t}\} \quad (12)$$

The factor $\frac{1}{\sqrt{N}}$ normalizes the signal energy, because we assume that the latter is not affected by the operation of OFDM modulation.

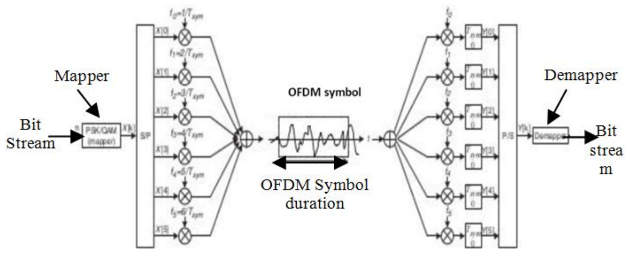


Figure 2. OFDM modulation and demodulation

Let f_c the center frequency of the signal as $f_c = f_0 + \frac{N}{2Ts}$ it goes back to write the above equation:

$$s(t) = \Re\left\{\Pi(t) e^{2\pi i f_c t} \sum_{k=0}^{N-1} \frac{x_k}{\sqrt{N}} e^{2\pi i t(k-N/2)/Ts}\right\} \quad (13)$$

We may write:

$$s(t) = \Re\{\Pi(t) s'(t) e^{2\pi i f_c t}\} \quad (14)$$

where $s'(t)$ is the complex envelope of the signal s(t) before the door windowing function. Its range is limited to the interval [-N/2Ts, N/2Ts], the signal s(t) can be sampled at a rate $\frac{\mathrm{N}}{\mathrm{Ts}}$ without there aliasing according to Shannon's theorem. The expression of the samples is obtained:

$$s'\left(\frac{nTs}{N}\right) = \sum_{k=0}^{N-1} \frac{x_k}{\sqrt{N}} e^{\frac{2\pi i n\left(k-\frac{N}{2}\right)}{N}} \quad (15)$$

$$= (-1)^n \sum_{k=0}^{N-1} \frac{x_k}{\sqrt{N}} e^{\frac{2\pi i n k}{N}} \quad (16)$$

$$= (-1)^n \, TFD^{-1}\{x_k\sqrt{N}\}_{k=0}^{N-1} \quad (17)$$

This result shows that the OFDM signal can be easily generated using the inverse discrete Fourier transform (DFT). Upon reception, the transmitted symbols can be found by applying the same way a direct discrete Fourier transform for samples received. Note that the algorithms of direct fast Fourier transform (FFT) and inverse (IFFT) allow an efficient implementation of DFT. The term $(-1)^n$ result of the simplification of the frequency shift term $e^{-i\pi n}$. This can translate the signal [0, N-1/T_s] to [-N/2Ts, N/2Ts]. This is seen as a shift of the spectrum around zero frequency. At output of the IFFT, the analytical signal OFDM baseband is recovered [5].

VII. OFDM Guard Interval

A simple method to reduce ISI is to increase the number N of subcarriers to increase the symbol duration T_s. However, the duration of each OFDM symbol should remain well below the coherence time of the channel and thus the total cancellation of the ISI by this method is not feasible. One solution is to deliberately sacrifice a portion of the energy emitted by inserting between each OFDM symbol a guard whose role is to absorb the residual ISI. The guard interval is a period during which, no useful data is transmitted. Duration must be greater than the maximum delay spread of the channel impulse response, the useful part of duration T_s of each OFDM symbol will be unaffected by the ISI. After insertion of the guard interval, the spacing between the subcarriers is equal to $\Delta f = \frac{1}{Ts}$ when the OFDM symbol duration is increased to $Ts' = T_s + T_\Delta$ causing a loss of orthogonality between subcarriers. This orthogonality can be restored with the proviso that during reception the rectangular windowing duration T_s on which the FFT is applied, the number of periods of the sinusoidal signals of each component of the OFDM signal must be integer. There are techniques to restore the orthogonality as the "cyclic prefix" and the "Zero-padding".

VIII. Cyclic Perfix

The OFDM guard interval can be inserted in two different ways. One is the zero padding that pads the guard interval with zeros. The other is the cyclic extension of the OFDM symbol with cyclic prefix or cyclic suffix. This case is shown in the Fig. 3. CP is to extend the OFDM symbol by copying the last samples of the OFDM symbol into its front.

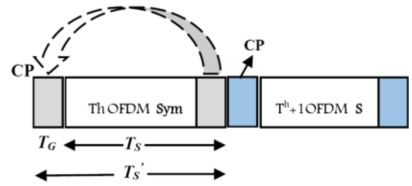


Figure 3. OFDM symbols with CP

The OFDM transceiver system including cyclic prefix is shown in Fig. 4.

It can be seen that if the length of the guard interval is set longer than or equal to the maximum delay of a multipath channel, the ISI effect of an OFDM symbol on the next OFDM is confined within the guard interval so that it may not affect the FFT of the next OFDM symbol taken for the duration of T_s. This implies that the guard interval longer than the maximum delay of the multipath channel allows for maintaining the orthogonally among the subcarriers. As the continuity of each delayed subcarrier has been warranted by the CP, its orthogonality with all other subcarriers is maintained over T_s [6].

$$\frac{1}{Ts} \int_0^{Ts} e^{2\pi i f_k (t-t0)} e^{-2\pi i f_i (t-t0)} \, dt = 0 \quad k \neq i \quad (18)$$

The OFDM receiver takes the FFT of the received samples $\{y[n]\}_{n=0}^{N-1}$ to yield:

$$Y[k] = \sum_{n=0}^{N-1} y[n] \, e^{-2\pi i k n/N} \quad (19)$$

$$= \sum_{n=0}^{N-1} \{\sum_{m=0}^{\infty} h[m] x[n-m] + n[n]\} \, e^{-2\pi i k n/N} \quad (20)$$

$$= \sum_{n=0}^{N-1} \left\{ \sum_{m=0}^{\infty} h[m] \sum_{n=0}^{N-1} \left\{ \frac{1}{N} \sum_{i=0}^{N-1} X[i] e^{\frac{2\pi i k(n-m)}{N}} \right\} \right\} . e^{-2\pi i k n/N} + N[k] \quad (21)$$

$$= H[k] X[k] + N[k] \quad (22)$$

These identities imply that the OFDM system can be simply thought of as multiplying the input symbol by the channel frequency response in the frequency domain. In other words, insertion of CP in the transmitter makes the transmit samples circularly convolved with the channel samples. Under no noise condition, the transmitted symbol can be detected by one tap equalization, which simply divides the received symbol by the channel.

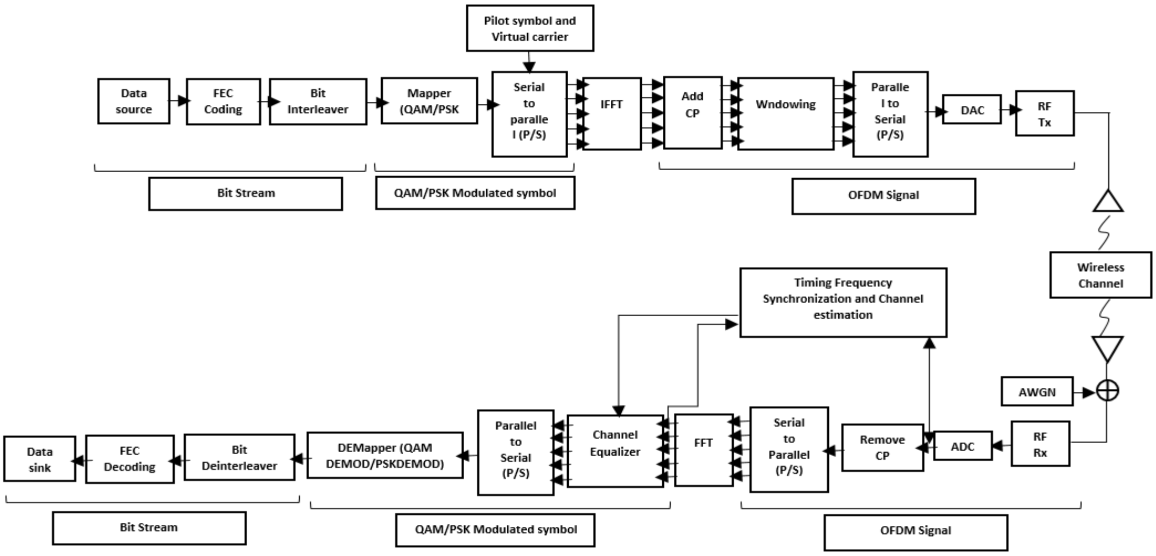


Figure 4. Block diagram of OFDM transceiver system

IX. Zero Padding

This particular approach is adopted by inserting zero into the guard interval. Even with the length of ZP longer than the maximum delay of the multipath channel, a small STO causes the OFDM symbol of an effective duration to have a discontinuity within the FFT window and therefore, the guard interval part of the next OFDM symbol is copied and added into the head part of the current symbol to prevent ICI. Since the ZP is filled with zeros, the actual length of an OFDM symbol containing ZP is shorter than that of an OFDM symbol containing CP or CS and accordingly, the length of a rectangular window for transmission is also shorter, so that the corresponding since-type spectrum may be wider. This implies that compared with an OFDM symbol containing CP or CS, an OFDM symbol containing ZP has PSD (Power Spectral Density) with the smaller in-band ripple and the larger out-of-band power, allowing more power to be used for transmission with the peak transmission with the peak transmission power fixed [6].

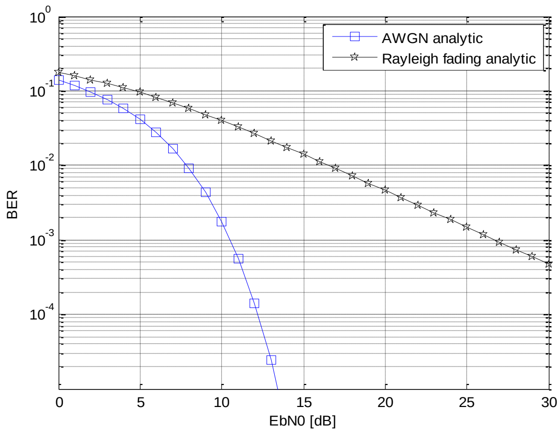
TABLE I. Fundamental Simulation Parameters

Parameters	Value
Number of Frames	3
FFT size	64
Virtual carriers	16
Used subcarriers	48
Modulation	16-QAM
Maximum Delay	15
Channel	AWGN/Rayleigh

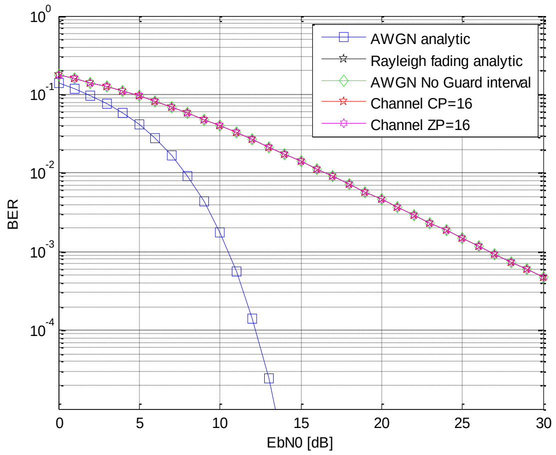
X. Simulation Results

1n this section we validate the BER performance of the OFDM systems associated to the cases when the guard interval is a cyclic prefix, cyclic suffix or Zero padding. The next Table I summarizes major OFDM system parameters for performance evaluation. We simulate the effect of ISI as the length of a guard interval varies. It consider the BER performance of an OFDM system with 64 points FFT and 16 virtual carriers, for 16-QAM signaling in the AWGN or a multipath Rayleigh fading channel Fig. 5(A).

It is clear from Fig. 5(B) and Fig. 5(C) that the BER performance with CP or ZF of length 16 samples is consistent with that of the analytic result in the Rayleigh fading channel. This implies that the OFDM system is just subject to a flat fading channel as long as CP or ZF is large enough. It is also clear that the BER performance in an AWGN channel is consistent with the analytical results. This is true regardless of how long GI is, because there is no multipath delay in the AWGN channel. However, the effect of ISI on the BER performance becomes significant in the multipath Rayleigh fading channel as the length of GI decreases, which eventually leads to an error floor.



(A)



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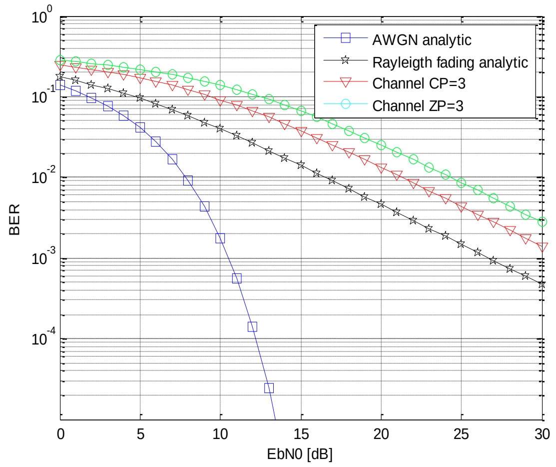

(C)

Figure 5. BER performance of conventional OFDM/16-QAM systems: (a) analytical channel (b) BER Performance for guard interval length=16 (C) BER Performance for guard interval length=3.

XI. Conclusion

This article contains as an essential result, confirming the possibility to introduce on OFDM system, a guard interval without causing a loss of orthogonality introduced on different carriers of the system highlighted. We suggest the presence of different types of guard interval, each of which contains its personal characteristic, and where we have described the simulation of these last. We can say at the end that the OFDM communication system combining orthogonality carrier and the introduction of the guard interval, to be considered as a promising solution for the implementation of the systems whose purpose is to reduce the complexity of design.

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