# Evaluating Accuracy of Fault Location Algorithms Based on Terminal Current and Voltage Data

Alkım Çapar and Ayşen Basa Arsoy Department of Electrical Engineering, Kocaeli, Turkey Email: {alkim.capar, aba}@kocaeli.edu.tr

Abstract—Accurate estimation of fault location is essential for reliability of power system. For this purpose, fault location algorithms are studied and compared in this paper. These algorithms are based on the measurements taken from either sending end of a transmission line or both sending and receiving ends of the line. The types of fault location algorithms are discussed first. The effectiveness of the algorithms are tested on a faulty transmission line simulated in DigSILENT. Algorithms developed in MATLAB use the voltage and current values obtained from the simulated system. The impact of the fault distance, fault types and the fault resistance are examined in detecting the fault location. The study results reveal that the symmetrical fault location algorithm accurately estimates the fault location in all cases.

*Index Terms*—one-end fault location, two-end fault location, fault location algorithms, fault resistance

## I. INTRODUCTION

When any kind of faults occur in a power system, the first action must be to clear the fault from the system. Once the protection action is taken, the most accurate distance of fault information should be provided to aid the user in locating the fault to remove the cause of the fault. Fault location can be estimated from current and voltages measured from one-end or two-end of the line.

Many studies are carried out on the fault location algorithms in transmission systems, including travelling wave algorithms [1]-[3], one-end fault location algorithms [4]-[6], two-end and multi-end fault location algorithms [7], [8]. Some parameters such as fault types, fault impedance and source impedance affect the performance of fault location algorithms.

Travelling wave algorithms have been used for fault location since 1931 for overhead lines and underground cables [9], [10]. These algorithms are improved to immune to fault type, fault resistance, fault inception angle and source parameters of a system [11]. However, high sampling rate, sampling window selection and problems of distinguishing between travelling waves reflected from a fault and the remote end of the line are drawbacks of travelling wave algorithms [12].

Drawbacks mentioned above yield to the development of one-end and two-end fault location algorithms. Oneend fault location algorithms are simple, economic and use only measurements from one terminal. On the other hand, the feature of one-end measurement may cause estimation errors. Two-end and multiple-end fault location algorithms based on synchronized [13] and unsynchronized [14] measurements of two or more bus terminals are utilized for higher accuracy. These algorithms can be developed considering various situations in a power system, for example load oriented power systems [15], a power system including FACTS [16], a power system having untransposed lines [17]. Solution skills of artificial intelligence can be an alternative method for fault location applications [18], [19].

In this paper, selected one-end and two-end fault location algorithms are described. The effects of the fault types, fault resistance and fault distance on the accuracy of these algorithms are demonstrated followed by comparison and discussion of the results.

## II. ONE-END FAULT LOCATION ALGORITHMS

One-end fault location algorithms usually use variables from the sending end. Fig. 1 is a basic circuit for fault location. Sending end voltage can be defined as;

$$V_S = I_S(mZ_L) + I_f R_f \tag{1}$$

where, *m*: distance to fault location,  $R_{j}$ : fault resistance,  $I_{j}$ : fault current,  $Z_{L}$ : line impedance,  $V_{S}$  and  $I_{S}$ : voltage and current at the sending end bus respectively.

As given in Fig. 1,  $V_R$  and  $I_R$  are voltage and current at the receiving end bus respectively and  $V_f$  is the fault voltage.



Figure 1. Circuit representation of line fault.

©2015 Engineering and Technology Publishing doi: 10.12720/ijeee.3.3.202-206

Manuscript received December 10, 2013; revised June 10, 2014.

#### A. Simple Reactance Method

This method compares the measured line impedance  $(Z_L)$  and calculated impedance  $(V_S/I_S)$  to find the fault location. Accuracy of this method depends on the angle of  $I_S$  being equal to the angle of  $I_f$ .

The simple form of the distance to the fault can be obtained as given in Eq. (2) where the fault resistance is ignored in Eq. (1), dividing this equation by  $I_s$  and saving the imaginary part;

$$m = Im(V_S/I_S)/Im(Z_L)$$
(2)

## B. Takagi Method

The Takagi method requires additionally pre-fault current values. This method improves simple reactance method by reducing the effect of load flow and minimizing the effect of fault resistance.

Superposition current (Isup) can be described as follows;

$$Isup = I - Ipre = If/d$$
(3)

where, I: fault current and Ipre: pre-fault current.

If the source and line have the same impedance, d becomes a real number. Accuracy of this method depends on this assumption.

Through Eq. (3) and Eq. (1);

$$V_S = I_S(mZ_L) + I_{sup} dR_f \tag{4}$$

$$V_{Sr} = mR_L I_{Sr} - mX_L I_{Si} + I_{supr} dR_f$$
(5)

$$V_{Si} = mX_L I_{Sr} + mR_L I_{Si} + I_{supi} dR_f$$
(6)

By multiplying Eq. (5) with  $I_{supi}$  and Eq. (6) with  $I_{supr}$  and subtract Eq. (6) from Eq. (5);

$$m = a/(b-c) \tag{7}$$

where;

$$a = V_{Sr} I_{supi} - V_{Si} I_{supr} \tag{8}$$

$$b = R(I_{Sr}I_{supi} - I_{Si}I_{supr})$$
(9)

$$c = X(I_{Sr}I_{supr} + I_{Si}I_{supi}) \tag{10}$$

## C. Modified Takagi Method

Modified Takagi method replaces superposition current with zero sequence current of sending end.

This method is limited with ground faults since zero sequence current exists for ground faults. Then, the fault distance is calculated as follows:

$$m = Im(3V_{S}I_{0}^{*}e^{-jT})/Im(3Z_{L}I_{S}I_{0}^{*}e^{-jT})$$
(11)

where,  $I_{0S}$ : zero sequence current and *T*: angle between  $I_{0S}$  and  $I_{f}$ .

## III. TWO-END FAULT LOCATION ALGORITHMS

Two-end fault location algorithms calculate fault location from the impedance seen from both end of the line. Because of accurately detecting fault location, these algorithms usually better than one-end fault location algorithms. Two-end fault location algorithms take  $V_f$  as a reference point.

#### A. Basic Two-End Method

This method is fundamental for the other two-end fault location methods.

$$V_f = V_S - I_S m Z_L \tag{12}$$

$$V_f = V_R - I_R(1 - m)Z_L$$
 (13)

Fault location can be calculated with Eq. (12) and Eq. (13);

$$m = (V_S - V_R + Z_L I_R) / (Z_L (I_S + I_R))$$
(14)

#### B. Symmetrical Fault Method

This method calculates the fault resistance with high accuracy for symmetrical faults and doesn't require line parameters for detecting fault location. Accuracy of this method depends on the fault type.

Voltage equations of  $V_S$ ,  $V_R$  and  $V_f$  can be derived from the circuitry given in Fig. 2 where D is the line length, 1 is the distance from the fault point and z is the line impedance.



Figure 2. Circuit representation of line fault for symmetrical fault method.

$$V_S - z l I_S - V_F = 0 \tag{15}$$

$$V_R - z(D - 1)I_R - V_F = 0$$
(16)

$$V_f = R_f (I_R + I_S) \tag{17}$$

With Eq. (15), Eq. (16) and Eq. (17), line parameters can be obtained;

$$z(D-l) = (V_R/I_R) - ((I_S+I_R)/I_R)R_f$$
(18)

$$zl = (V_S/I_S) - ((I_S + I_R)/I_S)R_f$$
(19)

$$zl = (a_1 + b_1 R_f) + j((a_2 + b_2 R_f)$$
(20)

$$z(D-l) = (c_1 + d_1R_f) + j((c_2 + d_2R_f)$$
(21)

where;

$$a_1 = Re(V_S/I_S) \tag{22}$$

$$a_2 = Im(V_S/I_S) \tag{23}$$

$$b_1 = Re(-((I_S + I_R)/I_S))$$
 (24)

$$b_2 = Im(-((I_S + I_R)/I_S))$$
 (25)

$$c_1 = Re(V_R/I_R) \tag{26}$$

$$c_2 = Im(V_R/I_R) \tag{27}$$

$$d_1 = Re(-((I_S + I_R)/I_R))$$
(28)

$$d_2 = Im(-((I_S + I_R)/I_R))$$
(29)

Z impedance is in first quadrant;

$$a_1 + b_1 R_f > 0$$
 (30)

$$a_2 + b_2 R_f > 0$$
 (31)

$$c_1 + d_1 R_f > 0$$
 (32)

$$c_2 + d_2 R_f > 0 \tag{33}$$

Both equations are part of the same line. Because of that, their angles must be equal;

$$\tan(\theta) = (a_2 + b_2 R_f) / (a_1 + b_1 R_f)$$
(34)

$$\tan(\theta) = (c_2 + d_2 R_f) / (c_1 + d_1 R_f)$$
(35)

From Eq. (34) and Eq. (35),  $R_f$  can be derived;

$$R_f^2 + pR_f + q = 0 (36)$$

where;

$$p = (a_1d_1 + b_1c_2 - a_2d_1 - b_2c_2)/(b_1d_2 - b_2d_1)$$
(37)

$$q = (a_1c_2 - a_2c_1)/(b_1d_2 - b_2d_1)$$
(38)

Eq. (36) has two roots;

$$(R_f)_1 = (-p + \sqrt{(p^2 - 4q)})/2 \tag{39}$$

$$(R_f)_2 = (-p - \sqrt{(p^2 - 4q)})/2 \tag{40}$$

Only one of roots gives the correct angle value;

$$\theta_1 = \arctan[(a_2 + b_2 R_{f1})/(a_1 + b_1 R_{f1})]$$
(41)

$$\theta_2 = \arctan[(a_2 + b_2 R_{f2})/(a_1 + b_1 R_{f2})]$$
(42)

Correct value of  $R_f$  should give an angle which is closer to typical values of the line characteristic and angle must be in the first quadrant.

Fault location can be given as;

$$1\% = 100(zl)/((zl+z(D-1)))$$
 (43)

By including Eq. (20), Eq. (21) and the correct fault resistance obtained from Eq. (41) or Eq. (42), the final form of the fault location is as follows;

$$1\% = 100(e/f)$$
 (44)

where;

$$e = (a_1 + b_1 R_f) + j(a_2 + b_2 R_f)$$
(45)

$$f = e + (c_1 + d_1 R_f) + j(c_2 + d_2 R_f)$$
(46)

#### IV. EVALUATION OF ALGORITHMS

The single line diagram of an 8 bus system that fault location algorithms are tested is shown in Fig. 3. The system consists of 3 synchronous generators, 2 external grids, 6 transformers and 2 asynchronous motors.



Fault location algorithms are tested at line L2 in Fig. 3, between bus 3 (sending end) and bus 4 (receiving end). Line parameters are given in Table I while bus parameters for bus 3 and 4 at load flow state are tabulated in Table II.

TABLE I. LINE PARAMETERS

	R	Х	С			
Positive						
sequence	0.03Ω/km	0.242Ω/km	14.86nF/km			
Negative						
sequence	0.03Ω/km	0.242Ω/km	14.86nF/km			
Zero						
sequence	0.131Ω/km	0.855Ω/km	6.44nF/km			
The length of transmission line is 100km						

TABLE II. BUS PARAMETERS

	Bus 3	Bus 4
Voltage	110.36kV	110.19kV
Frequency	50Hz	50Hz

Firstly, the algorithms are implemented on the system for phase to ground fault at different locations (Table III). Test distances are selected to be 30km, 50km and 70km from the sending end of the line.

TABLE III. A-Phase to Ground Fault at Different Locations (RF=0  $\Omega$ )

	30	%	50	%	70	%
	km	error	km	error	km	error
Takagi Method	30.5		50.91		71.31	
	km	1.66	km	1.82	km	1.87
Modified Takagi	30.51		50.92		71.31	
Method	km	1.7	km	1.84	km	1.87
Simple Reactance	30.51		50.92		71.31	
Method	km	1.7	km	1.84	km	1.87
Basic Two-End	29.93		50.06		70.17	
Method	km	0.23	km	0.12	km	0.24
Symmetrical	29.95		50.03		70.03	
Fault Method	km	0.16	km	0.06	km	0.04

As expected, when distance between fault location and sending bus increases, accuracy of one-end fault location algorithms decreases. The accuracy of these algorithms seems to be nearly the same, however the Takagi method has a less estimation error among them by reducing the effect of load flow. Two-end fault location algorithms show better performance in estimating the fault location with an error less than 1%. The best fault location estimation is obtained with the symmetrical fault method.

Secondly, the selected algorithms are tested for different faults at 50km (Table IV) fault distance from the sending end of the line.

TABLE IV. DIFFERENT FAULTS AT 50KM (RF= $0\Omega$ )

	a-b	%	b-c-t	%	a-b-c	%
		error		error		error
Takagi Method	52.8		51		51	
	km	5.6	km	2	km	2
Modified			51			
Takagi Method	-	-	km	2	-	-
Simple						
Reactance	52.8		51		51	
Method	km	5.6	km	2	km	2
Basic Two-End	50.3		50.2		50.2	
Method	km	0.6	km	0.4	km	0.4
Symmetrical	50.1		49.9		49.9	
Fault Method	km	0.2	km	0.2	km	0.2

One-end fault location algorithms exceed 2% error rate. On the other hand, two-end algorithms have maximum 0.6% error rate. The symmetrical fault method has the best accuracy rating.

Thirdly, algorithms are tested for asymmetrical faults with the fault resistance of 0.5, 1 and 1.5 ohm at the 50km fault distance from the sending end of the line (Table V).

 TABLE V.
 A-Phase to Ground Fault at 50km with Different Fault Resistances

	0.5	%	1Ω	%	1.5	%
	Ω	error		error	Ω	error
Takagi	51.1		51.3		55.2	
Method	km	2.2	km	2.6	km	10.44
Modified						
Takagi	51		51.1		55.2	
Method	km	2	km	2.2	km	10.44
Simple						
Reactance	51		51.1		55.2	
Method	km	2	km	2.2	km	10.44
Basic Two-	50		50		49.8	
End Method	km	0	km	0	km	0.4
Symmetrical	49.8		49.8		49.4	
Fault Method	km	0.4	km	0.4	km	1.2

The errors for one-end fault location algorithms are around 10%. Superposition current seems to have a negative effect on the Takagi method. Therefore, the accuracy of this method is worse than other methods. The estimation error obtained from the modified Takagi method and simple reactance method are the same. As expected the basic two-end method has better accuracy at an asymmetrical fault with the fault resistance.

Finally, algorithms are tested at symmetrical fault with the fault resistance as above at the 50km fault distance from the sending end of the line (Table VI).

	0.5 Ω	%	1Ω	%	1.5 Ω	%
		error		error		error
Takagi	50.82		49.83		50.46	
Method	km	1.64	km	0.34	km	0.92
Modified						
Takagi						
Method	-	-	-	-	-	-
Simple						
Reactance	50.82		49.83		50.46	
Method	km	1.64	km	0.34	km	0.92
Basic						
Two-End	50.18		49.88		50.03	
Method	km	0.36	km	0.24	km	0.06
Symmetric						
al Fault	50.09		49.88		49.99	
Method	km	0.18	km	0.24	km	0.02

TABLE VI. THREE PHASE FAULT AT 50KM WITH DIFFERENT FAULT RESISTANCES

The symmetrical fault method seems to be more accurate in this case because symmetrical fault has no zero sequence components affecting the calculation of the fault resistance.

In two cases, one-end location algorithms exceed 5% error rate causing more than 2km miscalculation. If the faults occur at a distance greater than 50km from the sending end, error rate will increase. This is an unwanted result for one-end fault location algorithms. Two-end fault location algorithms have maximum 0.6 km miscalculation in one case and other cases have less than 0.3km miscalculation. Two-end fault location algorithms generally have a better accuracy than one-end location algorithms and this test system proves that assumption.

### V. CONCLUSION

This paper evaluates the effects of certain parameters on the accuracy of fault location algorithms. These algorithms use voltage and current data taken from one or two end of transmission lines. The developed algorithms are implemented on a test system.

One-end fault location methods give nearly identical results. The modified Takagi method does not use prefault values like the simple reactance method except this method is limited to only ground faults while the simple reactance method can be used at all fault types with the same accuracy. The Takagi method uses pre-fault current, which requires extra calculation and measurement values. Though these measurements and calculations do not improve its accuracy because of the source and line impedance difference. The simple reactance method could be a better choice since it has the same accuracy in handling the problem of impedance difference between the source and the line without requiring extra components.

The symmetrical fault method gives better results except for asymmetrical faults with a fault resistance in two-end fault location algorithms. Basic two-end method has the best results at asymmetrical fault with a fault resistance. Both of these methods have acceptable accuracy for any fault, but if the fault type is predetermined and the suitable two-end fault location method is chosen, then the fault location estimation error is minimized.

#### REFERENCES

- B. Xu, "Fault location technology of transmission lines based on traveling waves," Ph.D. dissertation, Dept. Elec. Eng., Xi'an Jiaotong University, Xi'an, 1991.
- [2] P. Chen, B. Xu, and J. Li, "Modern traveling wave based fault location technology and its application," *Automation of Electric Power Systems*, vol. 25, no. 23, pp. 62-65, 2001.
- [3] X. J. Zeng, N. Chen, Z. W. Li, and F. Deng, "A novel algorithm for traveling wave fault location base on network," in *Proc. IEEE International Conference on Industrial Technology*, April 21-24, 2008, pp. 1-5.
- [4] T. Takagi, Y. Yamakoshi, M. Yamaura, R. Kondow, and T. Matsushima, "Development of a new type fault locator using the one-terminal voltage and current data," *IEEE Trans. Power App. Syst.*, vol. PAS-101, no. 8, pp. 2892-2898, Aug. 1982.
- [5] Q. Zhang, Y. Zhang, W. Song, and Y. Yu, "Transmission line fault location for phase-to-earth fault using one-terminal data," *IEEE Proc.-Gener. Transm. Distrib.*, vol. 146, no. 2, pp. 121-124, Mar. 1999.
- [6] Z. Qingchao, Z. Yao, S. Wennan, Y. Yixin, and W Zhigang, "Fault location of two-parallel transmission line for non-earth fault using one terminal data," *IEEE Trans. Power Del.*, vol. 14, no. 3, pp. 863-867, Jul. 1999.
- [7] D. A. Tziouvaras, J. Roberts, and G. Benmouyal, "New multiended fault location design for two- or three-terminal lines," in *Proc. Seventh International Conference on Developments in Power System Protection*, April 9-12, 2001, pp. 395-398.
- [8] G. E. Alexander and J. M. Kennedy, "Evaluation of Phasor-Based Fault Location Algorithm," in GE Protection and Control, Malvern, SAD, 1996, pp. 1-11.
- [9] L. J. Lewis, "Travelling wave relations applicable to power system fault locators," *AIEE Transactions*, vol. 70, no. 2, pp. 1671-1680, Jul. 1951.
- [10] P. F. Gale, P. A. Crossley, X. Bingyin, Y. Ge, B. J. Cory, and J. R. G. Barker, "Fault location based on travelling waves," in *Proc. Fifth International Conference on Developments in Power System Protection*, 1993, pp. 54-59.
- [11] M. Silva, M. Oleskovich, and D. V. Coury, "A fault locator for transmission lines using travelling waves and wavelet transform theory," in *Proc. Eighth IEE International Conference on Developments in Power System Protection*, April 5-8, 2004, pp. 212-215.
- [12] M. M. Saha, J. Izykowski, and E. Rosolowski, Fault Location on Power Networks, London: Springer-Verlag, 2010.
- [13] G. Preston, Z. M. Radojevic, C. H. Kim, and V. Terzija, "New settings-free fault location algorithm based on synchronized sampling," *Generation, Transmission and Distribution, IET*, vol. 5, no. 3, pp. 376-383, Mar. 2011.
- [14] J. Izykowski, E. Rosolowski, P. Balcerek, M. Fulczyk, and M. M. Saha, "Accurate noniterative fault-location algorithm utilizing

two-end unsynchronized measurements," *IEEE Transactions on Power Delivery*, vol. 26, no. 2, pp. 547-555, Apr. 2011.

- [15] C. E. de Morais Pereira and L. C. Zanetta, "Fault location in multitapped transmission lines using unsynchronized data and superposition theorem," *IEEE Transactions On Power Delivery*, vol. 26, no. 4, pp. 2081-2089, Oct. 2011.
- [16] M. Ghazizadh-Ahsaee and J. Sadeh, "Accurate fault location algorithm for transmission lines in the presence of shuntconnected flexible AC transmission system devices," *Generation*, *Transmission and Distribution, IET*, vol. 6, no. 3, pp. 247-255, Mar. 2012.
- [17] D. A. G. Vieira, D. B. Oliveira, and A. C. Lisboa, "A closed-form solution for untransposed transmission-lines fault location with nonsynchronized terminals," *IEEE Transactions On Power Delivery*, vol. 28, no. 1, pp. 524-525, Jan. 2013.
- [18] R. K. Aggarwal, S. L. Blond, P. Beaumont, G. Baber, F. Kawano, and S. Miura, "High frequency fault location method for transmission lines based on artificial neural network and genetic algorithm using current signals only," in *Proc. 11th International Conference on Developments in Power Systems Protection*, April 23-26, 2012, pp. 1-6.
- [19] G. K. Purushothama, A. U. Narendranath, D. Thukaram, and K. Parthasarathy, "Ann applications in fault locators," *International Journal of Electrical Power and Energy Systems*, vol. 23, no.6, pp. 491-506, Aug. 2001.



Alkım Çapar was born in Balıkesir, Turkey on June 1988. Alkım Çapar graduated from Kocaeli Universty in 2012. Currently, Alkım Çapar is a Research Assistant and Master Student in Department of Electrical Engineering at Kocaeli University, in Kocaeli, Turkey.



Ayşen B. Arsoy received her B.Sc. Degree from Istanbul Technical University, Turkey, in 1992, M.S. degree from the University of Missouri-Rolla in 1996, and Ph.D. degree from Virginia Polytechnic Institute and State University in 2000, all in electrical engineering. She is currently an Associate Professor at Kocaeli University. Her research interests include power system

modeling and analysis, power system protection, distributed generation, energy

storage and power electronics applications in power system.