Feedback Error Based Discontinuous and Continuous Variable Learning Rate CMAC

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***Abstract*—In this paper, Cerebellar Model Articulation Controller (CMAC) is used in conjunction with feedback control law to control a second order unstable plant. The memory of CMAC is updated according to the feedback error which makes the learning rate of CMAC sensitive to feedback error and hence responsible for the convergence of error. The paper starts with the comparison of unit step response of second order plant for different learning rates which show that the proposed variation in CMAC is sensitive to learning rates. Performance of the response is measured in terms in terms of key characteristics like rise time, peak overshoot, settling time and integral square error. Then discontinuous and continuous variable learning rate schemes are proposed to ameliorate the response of the plant in terms of peak overshoot, settling time and integral square error which is well supported by the simulation results done in MATLAB Simulink.**

***Index Terms*—Cerebellar model articulation controller, discontinuous variable learning rate CMAC, continuous variable learning rate CMAC**

I. Introduction

The CMAC known as Cerebellar Model Articulation/Arithmetic Controller was proposed by J. Albus in 1975 [1], [2]. CMAC performs a multivariable function approximation in a generalized look-up table form. Due to its high learning speed and local generalization it is used in variety of applications [3]-[7]. In the memory update rule, learning rate is one of the key parameter of CMAC which is responsible for the rate of convergence of error. The effect of learning rate on the response of the controlled plant is illustrated with the help of second order unstable plant. The control law is derived using feedback control law in conjunction with CMAC. It is found that CMAC is sensitive to learning rates as learning rate decides the speed with which the response converges to zero error. It is evident that increase in learning rate is accompanied by the rapid convergence to zero error, but increasing learning rate in a quest to reach the zero error tends to make the system oscillatory. So learning rate must be chosen wisely so that response will be faster as well as there are no oscillations in the steady state.

The response of the system can be improved by changing the control law which usually demands high control effort. In order to improve the response of the system without changing the control effort appreciably there must be some modifications in the CMAC algorithm. Intuitively learning rates must be high when error is large and must be small when error is small. But in basic CMAC learning rate is fixed. Thus two variable learning rate schemes are suggested so as to improve the response of the system without changing the control effort appreciably. The simulation studies shows that the variable learning rate schemes are better than basic CMAC in terms of peak overshoot, settling time and Integral square error.

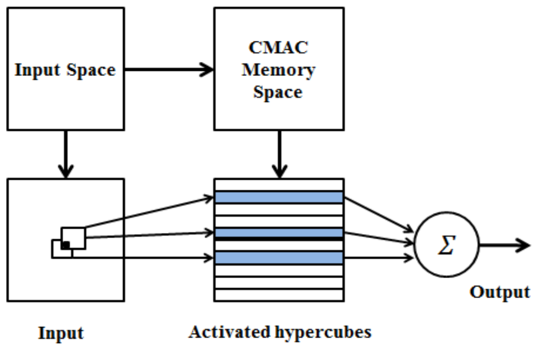

Figure 1. Cerebellar model articulation controller

II. Cerebellar Model Articulation Controller

CMAC is a learning structure which emulates the human cerebellum. It's an associative neural network [8]-[10] in which a small subset of the network influences any instantaneous output and that subset is determined by the input to the network as shown in Fig. 1. The region of operation of inputs is quantized say m inputs i.e. the number of elements in a particular input is m. This quantization determines the resolution of the network [11] and the shift positions of the overlapping regions. If n inputs are presented to the network, then total number of elements in input space is m^n which is quite large. To reduce this memory, inputs presented are converted into hyper cubes or hyper rectangles. A particular input vector is the sum of L nearby inputs i.e. a particular input is the overlapping of L nearby inputs. The number L is called the number of layer of CMAC which is referred to as the generalization width of CMAC. Thus m^n number of memory is converted into A memory units such that $A<<m^n$. The outline of CMAC algorithm is given below:

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(i) Number of inputs=n

(ii) Number of elements for a particular input=m, which is also the number of quantized state for input.

(iii) Total memory=m^n

(iv) Number of layers of CMAC=L

(v) Number of hypercubes in i^{th} layer=k_i, where i=1, 2, 3…..L

(vi) Total number of hypercube= $\sum k_i{}^n$, which is the memory of CMAC.

When an input is encountered, L hypercubes are activated and output of the network is simply the sum of the contents of that hypercube. That is if CMAC is an approximator its basis function can be defined as [11]

$$h_i = \begin{Bmatrix} 1, for\ activated\ hypercubes \\ 0, if\ not\ activated \end{Bmatrix} \tag{1}$$

The memory content update rule is given by least square mean as [1]

$$A(t) = A(t-1) + lr.e/L \tag{2}$$

where lr is the learning rate of CMAC and e is the network error.

In this paper, there is a slight modification in the weight update law by taking the error of the feedback system instead of network error and hence called feedback error based CMAC. The main focus of the paper is to study and observe the behavior of learning rate of CMAC and then suggest two variable learning rate schemes which improve the response of the system. The output value for an input point can be considered as a weighted sum of selected basis functions. The order of the plant determines the number of inputs of CMAC.

A. Effect of Learning Rate on CMAC

Increase in learning rate implies that the response will try to reach the zero error level as soon as possible. Thus the consequence of increasing learning rate is the faster rise time which can be seen from the simulation results and from the table. Though there must be some constraints on the learning rate of CMAC as higher learning rates may lead to the instability. As far as the stability concerned, higher learning rate means large value of overshoot but faster settling time while lower learning rates have sluggish response and have large settling time. Thus while choosing the values of learning rates there is a tradeoff between overshoot and settling time. Hence learning rate is one of the parameters of CMAC which needed to be chosen wisely. The variation of peak overshoot with learning rate is shown in Fig. 2. Though another way to get the desired response is to change the error dynamics of feedback control law but usually it leads to the larger control effort which is not desirable in control system applications. In order to improve the response of the system CMAC algorithms needs to be altered such that there is no appreciable change in feedback control law. Another way to improve the response of the system is to change the learning rate of the system according to error. That is, when error is high learning rate is made high so that it move towards the zero error quickly when error is low learning rate is made lower so that the response doesn't go beyond the zero error level due to higher learning rate. Thus to apply this logic response of the system is divided into four regions. In order to understand why these four regions are used one must understand the implications of using variable learning rate scheme. Initially the error is very high due to which larger learning rate is recommended. Due to higher learning rate, response will reach the zero error level quickly but it will not stop at this level and it will go further. When it goes beyond the prescribed level of error, learning rate is increased further due to which there is a dampening in the speed of response due to which the overshoot as compared to fixed learning rate scheme will decrease. As the error reaches within the prescribed range, learning rate can be made lower as there is no need to learn more. Not only this, due to the curbing of response in upward direction due to the higher learning rate response tends to move in downward direction quickly, the decrease in learning rate in this region tends to slow down the response in downward direction. In spite of the lowering in learning rate in this region it may be possible that undershoot may cross the prescribed error range. Thus again to suppress undershoots, learning rate must be increased such that it will push the response towards zero error position. To reduce the overshoot further, one may keep the learning rate very low at the start of the response due to which rise time will increase but it's harmless as compared to the peak overshoot. Thus response of the system is divided into four regions with different learning rate namely lr1, lr2, lr3 and lr4 respectively.

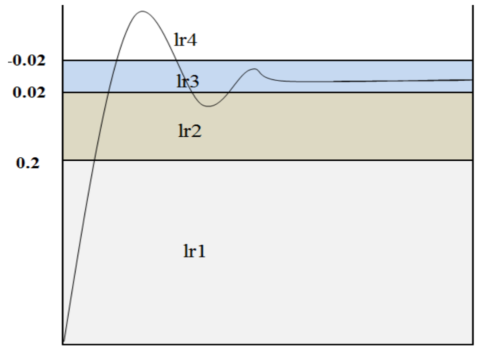

Figure 2. Division of unit step response region according to learning rates

III. Discontinuous Variable Learning Rate CMAC

In this variable learning rate scheme, the step response of a system is divided into four regions as shown in Fig. 2.

A. Region 1

In this region error is high so it's logical to make the learning rate high. But it is seen that the higher learning rate makes the response reaching to the zero error level quickly but at the same time due to higher learning rate response becomes very fast and that may lead to the higher peak overshoots. So it's advisable to keep the learning rate in this region to be low which obviously occurs at the cost of rising time.

B. Region 2

This region is decided on the basis of undershoots which occurs due to curbing of peak overshoot. In real time applications undershoot are undesirable. Learning rate in this region can be made high and low depending upon the two cases.

Case 1: If there are not any undershoots or undershoots not going beyond Region 3. Then the value of learning rate in this region must be made very low as it serves two purposes. First, at the start of the response lower value of learning rate slows the speed of response due to which peak overshoots in the region 4 will not be high. Second, as there are not undershoots after the peak overshoot, there is no chance that these undershoots are going to occur in future.

Case 2: In case the peak overshoots are very high in the region4 and to suppress that learning rate is made very high, there is a chance that suppression may push the response in downwards direction to such extent that it may cross the prescribed error level. Thus to suppress that overshoot learning rate is made high but not higher than lr4 in this region.

C. Region 3

In this region error is small so learning rate must be kept small so as to avoid the harmful effects of higher learning rate.

D. Region 4

Since error is high in this region, learning rate must be kept high in comparison to other three regions. This will curb the overshoots. Though making it much higher tends to push the response in downwards direction which may lead to undershoots as discussed in Region 2 section.

That is, learning rate

$$lr = \begin{Bmatrix} lr1, & 0.2 < e < 1 \\ lr2, & 0.02 < e < 0.2 \\ lr3, & -0.02 < e < 0.02 \\ lr4, & e < -0.02 \end{Bmatrix} \tag{3}$$

Thus, from the four regions it is evident that if first case of region 2 occurs then $lr1 < lr3 < lr2 < lr4$ else for the other cas $lr1, lr2 < lr3 < lr4$. Note that the learning rate is varied sharply with respect to error i.e. the variation of learning rate with respect to error is made discontinuous. To vary the learning rate smoothly with respect to error continuous variable learning rate CMAC is proposed.

IV. Continuous Variable Learning Rate CMAC

This variable learning rate scheme uses the same logic as that scheme1. But rather learning rate is not varied sharply but smoothly with respect to error.

Learning rate can be written as,

For case 1:

$$lr = \begin{Bmatrix} lr1 & ,0.2 < e < 1 \\ lr1\left(\frac{lr1}{lr4}\right)^{\frac{e-0.2}{0.18}} & ,0.02 < e < 0.2 \\ lr4\left(\frac{lr4}{lr3}\right)^{50(|e|-0.02)} & ,-0.02 < e < 0.02 \\ lr4(exp)^{5(e+0.02)} & ,e < -0.02 \end{Bmatrix} \tag{4}$$

For case 2:

$$lr = \begin{Bmatrix} lr1\left(\frac{lr1}{lr2}\right)^{1.25(e-1)} & ,0.2 < e < 1 \\ lr2\left(\frac{lr2}{lr4}\right)^{\frac{e-0.2}{0.18}} & ,0.02 < e < 0.2 \\ lr4\left(\frac{lr4}{lr3}\right)^{50(|e|-0.02)} & ,-0.02 < e < 0.02 \\ lr4(exp)^{5(e+0.02)} & ,e < -0.02 \end{Bmatrix} \tag{5}$$

V. Control Scheme

To demonstrate the behavior of learning rates simulations are done on unstable second order plant in which CMAC is used in conjunction with feedback control law to control the plant. Though CMAC has the capability to learn nonlinear functions quickly, linear plant is discussed here for simplicity. The plant can be written in state space form as

$$\begin{Bmatrix} \dot{x}_1 = x_2 \\ \dot{x}_2 = ax_1 + bx_2 + cu \end{Bmatrix} \tag{6}$$

It can also be written as

$$\begin{Bmatrix} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x_1, x_2) + cu \end{Bmatrix} \tag{7}$$

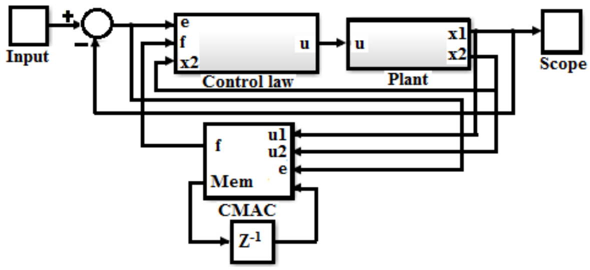

Figure 3. Simulink diagram of CMAC

Control law for the state space equation given by (8) can be written as,

$$u = \left[-k_1(x_1 - y_r) - k_2 x_2 - \hat{f}\right]/c \tag{8}$$

where y_r is the input signal, $\hat{f}$ being the function to be estimated online, k_1 and k_2 is such that the polynomial $s^2 + k_2 s + k_1$ is Hurwitz. The Simulink diagram to implement the CMAC is shown in Fig. 3.

VI. Simulation and Results

For simulation purpose, an unstable plant with transfer function as given by (6) is used

$$G(s) = \frac{10}{s^2+10s-20} \tag{9}$$

This plant can be written in state space form

$$\begin{Bmatrix} \dot{x}_1 = x_2 \\ \dot{x}_2 = 20x_1 - 10x_2 + 10u \end{Bmatrix} \tag{10}$$

Since the plant is of second order, two inputs CMAC is used. Parameters of two input CMAC

Quantization=20

Generalization=6

Number of hypercube=105

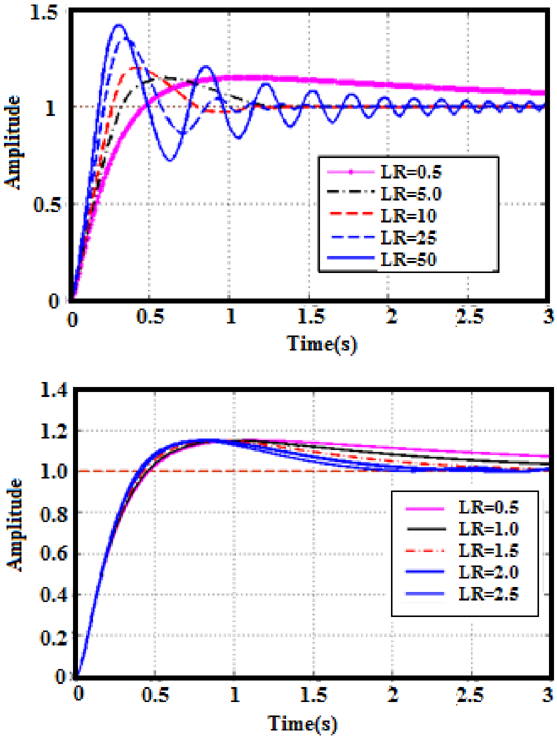

Figure 4. Response of the system for different learning rates

TABLE I. EFFECT OF DIFFERENT LEARNING RATES WHEN COVERING LARGE RANGE OF LEARNING RATE

Learning rate	Rise time(s)	Peak overshoot (%)	Settling time(s)	Integral Square Error
0.5	0.4735	14.98	5.7992	0.1603
5	0.3073	14.89	1.0162	0.1146
10	0.2592	23.65	1.0132	0.1127
20	0.2216	35.09	0.9772	0.1152
25	0.2119	39.34	3.1549	0.1225
30	0.2010	42.15	3.0742	0.1276
50	0.1783	48.85	4.0260	0.1488

TABLE II. EFFECT OF DIFFERENT LEARNING RATES WITH LOWER LEARNING RATES

Learning rate	Rise time(s)	Peak overshoot (%)	Settling time(s)	Integral Square Error
0.5	0.4735	14.98	5.7992	0.1603
1.0	0.4432	14.83	3.1054	0.1414
1.5	0.4182	14.73	2.2112	0.1338
2.0	0.3948	14.61	1.7882	0.1287
2.5	0.3753	14.53	1.5423	0.1250

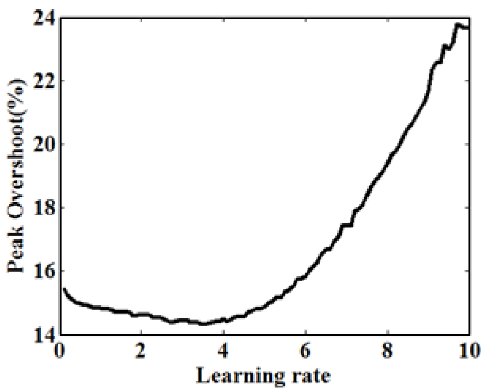

Figure 5. Variation of peak overshoots with learning rates

The range of inputs x_1 and x_2 is taken to be -5 to 5 units. From the Table I and Table II and simulation results of Fig. 4, it is concluded that peak overshoot increases with increase in learning rate. Though increasing in learning rate beyond 10 makes the response oscillatory due to which integral square error increases slightly. From the response of the system it is easy to see that for higher learning rate error though converges to zero value but it will oscillate around zero error level. If learning rate is taken to be low it is found that peak overshoot decreases with increase in learning rate. From Fig. 5, it has been found that peak overshoot remains nearly the same at about 15% for learning rate from 0 to 5. After that range it will increase monotonically with the increase in learning rates. So the response for the learning rates from 0.5 to 2.5 is chosen.

To improve the response two variable learning schemes are used. For the variable learning schemes to be implemented two cases which have been discussed in Region 2 must be taken into account. Case1 is applicable if there are no undershoots or the value of undershoots are very less. It is evident from the response that there are no undershoots for lower learning rates.

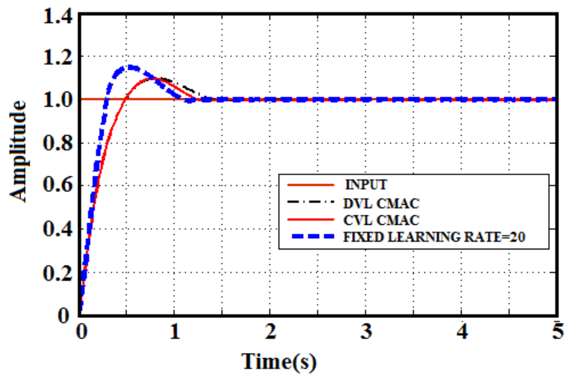

Figure 6. Comparison of unit step responses of CMAC, DVL CMAC and CVL CMAC

TABLE III. RESPONSE FOR DISCONTINUOUS VARIABLE LEARNING SCHEME

*lr*1	*lr*2	*lr*3	*lr*4	Rise time	Peak overshoot	Settling time	Integral square error
0.1	0.1	0.5	2.0	0.5009	11.57	1.9265	0.1329
0.1	0.1	0.5	3.0	0.5009	10.77	1.5706	0.1300
0.1	0.1	0.5	4.0	0.5009	10.19	1.3921	0.1284
0.1	0.1	0.5	5.0	0.5009	9.71	1.2817	0.1275
0.1	0.1	0.5	6.0	0.5009	9.37	1.2050	0.1269
0.1	0.1	0.5	7.0	0.5009	9.06	1.1468	0.1265
0.1	1.0	0.5	1.5	0.5025	12.68	2.3082	0.1369
0.1	1.5	1.0	2.0	0.4939	12.40	1.9232	0.1342
0.1	1.0	0.5	2.0	0.4963	12.11	1.9239	0.1339
0.1	1.5	0.5	2.0	0.4939	12.40	1.9233	0.1343
0.1	1.0	0.5	2.5	0.4963	11.66	1.7066	0.1318
0.1	1.5	0.5	2.5	0.4939	11.94	1.7050	0.1322
0.1	0.1	0.5	1.5	0.5010	12.12	2.3046	0.1359
0.1	0.1	0.5	2.0	0.5010	11.57	1.9272	0.1329
0.1	0.1	0.5	2.5	0.5010	11.14	1.7113	0.1312

TABLE IV. RESPONSE FOR CONTINUOUS VARIABLE LEARNING SCHEME

$lr1$	$lr2$	$lr3$	$lr4$	Rise time	Peak overshoot	Settling time	Integral square error
0.1	0.1	0.5	2.0	0.5000	11.14	1.6107	0.1304
0.1	0.1	0.5	3.0	0.5000	10.45	1.3620	0.1284
0.1	0.1	0.5	4.0	0.5000	9.95	1.2332	0.1274
0.1	0.1	0.5	5.0	0.5000	9.58	1.1515	0.1267
0.1	0.1	0.5	6.0	0.5000	9.28	1.0921	0.1263
0.1	0.1	0.5	7.0	0.5000	9.03	1.0486	0.1260
0.1	1.0	0.5	1.5	0.4786	13.20	2.1663	0.1360
0.1	1.5	1.0	2.0	0.4728	13.10	1.8029	0.1336
0.1	0.1	0.5	2.5	0.4998	11.23	1.6512	0.1309

Thus Case1 is applicable for lower learning rates. For simulation,

$$lr1 = lr2 = 0.1,\ lr3 = 0.5, lr4 = 5.$$

From Fig. 6 Peak overshoot gets reduced from 14.88% to 9.71% and ISE=0.1275. Settling time=1.2817 sec. Table III and Table IV show the response for variable learning rate scheme.

Case 2 is applicable for higher learning rates.

For Simulation,

$$lr1 = 0.1, lr2 = 10, lr3 = 0.5, lr4 = 20$$

If Case 2 arises, from Fig. 7 the continuous and discontinuous variable learning rate scheme shows significant improvement over single fixed learning rate of 20 by decreasing the peak overshoot of 34.7% to 10.53% and 11.93% respectively. Table V and Table VI show the different learning rates of the region for CVL CMAC and DVL CMAC. CVL CMAC is better than DVL CMAC in rise time, settling time and integral square error while peak overshoots of DVL CMAC is marginally better than that of CVL CMAC. From Fig. 3 the minimum value of overshoot for different learning rate is 14.33, the CVL and DVL scheme have been able to suppress the overshoots below 14.33 which implies both the CVL CMAC and DVL CMAC are better than fixed learning rate feedback error based CMAC in suppressing overshoots.

TABLE V. RESPONSE FOR DISCONTINUOUS VARIABLE LEARNING RATE SCHEME

$lr1$	$lr2$	$lr3$	$lr4$	Rise time	Peak overshoot /undershoots	Settling time	Integral square error
0.1	10	0.5	20	0.4652	10.53/4.69	1.2969	0.1261
0.1	10	0.5	25	0.4652	9.94/5.48	1.2864	0.1259
0.1	10	0.5	30	0.4652	9.49/6.19	1.2760	0.1259
0.1	10	5.0	20	0.4652	10.53/4.69	1.2923	0.1261
0.1	10	5.0	25	0.4652	9.94/5.45	1.2853	0.1259
0.1	10	5.0	30	0.4652	9.50/6.17	1.2747	0.1259

TABLE VI. RESPONSE FOR CONTINUOUS VARIABLE LEARNING RATE

$lr1$	$lr2$	$lr3$	$lr4$	Rise time	Peak overshoot /undershoots	Settling time	Integral square error
0.1	10	0.5	20	0.4046	11.93/6.98	1.0716	0.1242
0.1	10	0.5	25	0.4046	11.65/7.95	1.0308	0.1241
0.1	10	0.5	30	0.4046	11.45/8.77	1.0142	0.1242
0.1	10	5.0	20	0.4032	11.93/6.98	1.0718	0.1242
0.1	10	5.0	25	0.4032	11.64/7.96	1.0310	0.1241
0.1	10	5.0	30	0.4032	11.44/8.73	1.0139	0.1242

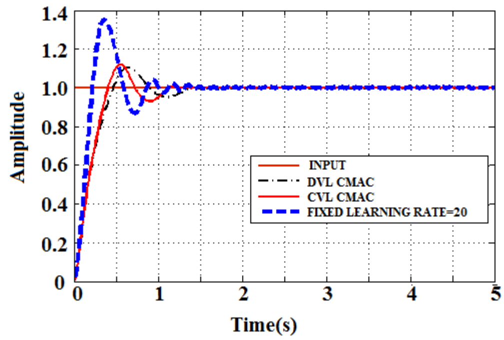

Figure 7. Comparison of unit responses of CMAC, DVL CMAC and CVL CMAC

VII. CONCLUSION

In a feedback error based CMAC, the response of the system is sensitive to learning rates as shown in Fig. 3. Lower learning rate gives sluggish response with low rise time and large integral square error while higher learning rate is accompanied by the fast response, low settling time and low integral square error though it may lead to the small amplitude oscillation in steady state. Fast response lead to the higher value of overshoots which is undesirable. The variation of peak overshoot with learning rate is shown in Fig. 3. To avoid the undesirable characteristics of fixed learning rate, learning rate is made to vary according to error. Fig. 1 shows the schematic of division of step response according to learning rate. Continuous variable learning rate (CVL) CMAC and Discontinuous variable learning (DVL) CMAC are the two variable learning rate schemes applied to curb the overshoots without affecting the settling time and integral square error. Two types of response are possible while applying the mentioned learning schemes. In the first type, the learning rate in the negative error region is not very high which leads to either no overshoots or small value of overshoot. Table III and Table IV show the response for various values of learning rate for this type of response. While in the second type, the learning rate in the negative error region is very high which leads to undershoots. Table V and Table VI show the response for various values of learning rate for this type of response. In both the type of response, CVL CMAC and DVL CMAC shows better performance than fixed learning rate CMAC in terms of peak overshoot. Fig. 4 and Fig. 5 shows that CVL CMAC and DVL

CMAC have better response than fixed learning rate CMAC. Rate of convergence of CVL CMAC is slightly better than that of DVL CMAC while DVL CMAC handles the overshoots better than CVL CMAC.

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