Improving IMC Method for BLDC Speed Control by Using Steady-State DC Model

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***Abstract*—This paper describes an improved method for controlling speed of Brushless DC motor (BLDC). Internal Model Control (IMC) method is used to achieve the good characteristics such as robustness, easiness, flexibility and stability. Here, this novel idea uses the steady-state DC model instead of BLDC's or general DC's to reduce large amount of calculation but it still achieves positive features of IMC method. To challenge the robustness, the proposed method is performed with not only nominal parameters but also large change of plant's parameters. On MatLAB Simulink, the obtained results show that the output signals well respond the referred signals. To be more objective, our results are compared with 2 previous approaches.**

***Index Terms*—Brushless DC (BLDC), Internal Model Control (IMC), speed control, Direct Current (DC) motor model-based, steady-state DC motor model-based, robust control**

I. Introduction

These days, Brushless DC motors have a wide range of applications in industrial automation, instrumentation, aerospace for several reasons. BLDC motors have no brush like DC's as well as slip like induction motors'; they use electronic commutation instead because they are permanent magnet synchronous motors. Based on these characteristics, BLDC motors are long life and noiseless operation, high dynamic, high efficiency, and large speed range. Since electronic commutation, the 3-phase input voltage has to be depended on rotor position which can be detected through 2 common ways, 3 Hall sensors [1]-[3] and back electromotive force (back-EMF) [4], [5]. The BLDC motor's back-EMF waveform can be sinusoidal [5], [6] or trapezoidal [2], [7], [8].

In some applications as computer's hard disk drivers or flying robot, the angular speed of BLDC motor need to be stable even during changes of environment or motor behaviors. For instance, friction coefficient may change after a time of operating or resistance may increase if the temperature increases. Moreover, the motor's data are usually incomplete. Hence, being stable under those uncertainties is called robustness [9]. There were many researches about robustness control such as model reference adaptive backstepping approach [10], auto-tunning algorithm [11], and neural net-based [12]. The Internal Model Control (IMC) is also a kind of robustness control which is introduced in 1980's by Garcia and Morari in chemical process control [13]. Nowadays, IMC method is widely applied beside chemical area because it is simply designed based on system model, well working with linear systems, and easy to control output response by using a first order low-pass filter. However, it has some problems with non-linear and Multi-Input Multi-Output (MIMO) systems; especially, there is great amount of calculation [14], [15]. References [16], [17] show a way to assist IMC method using phase-lock loop.

Our previous approach [18] described an idea to reduce the amount of calculation by using general DC motor model instead of BLDC's. With a higher step of improvement than [18], this paper now contributes a novel idea of using steady-state DC motor model instead of general DC's.

The remaining parts of this paper are arranged as follows. First, mathematical equations of BLDC model is presented in Section II. Then, Section III shows the construction of IMC system to control speed of BLDC motor. Next, in Section IV, the both general and steady state DC motor models are described to build Forward and Inverse Model of IMC system. After that, the comparison of robustness between the 2 IMC systems using BLDC model-based and steady-state DC model-based is shown in Section V. Finally, conclusions are indicated in Section VI.

II. Mathematical Equations of BLDC Model

A. *Mathematical Equations of BLDC Motor in Laplace Domain [8]*

The 3 phase currents i_a, i_b and i_c are calculated as follows:

$$\begin{cases} i_a = \dfrac{V_a - E_a - Ri_a}{Ls} \\ i_b = \dfrac{V_b - E_b - Ri_b}{Ls} \\ i_c = \dfrac{V_c - E_c - Ri_c}{Ls} \end{cases} \tag{1}$$

where:
V_a, V_b, V_c: Stator phase voltage;
R: Stator winding phase resistance;

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L: Stator winding phase effective inductance;
E_a, E_b, E_c: Phase back electromotive force (back-EMFs);
s: Laplace factor.

Additionally, the back-EMFs are given as:

$$\begin{cases} E_a = K_e \omega_m F(\theta_e) = K_e \omega_m F_a \\ E_b = K_e \omega_m F(\theta_e + \dfrac{2\pi}{3}) = K_e \omega_m F_b \\ E_c = K_e \omega_m F(\theta_e + \dfrac{4\pi}{3}) = K_e \omega_m F_c \end{cases} \quad (2)$$

where:
K_e: Back-EMF constant, voltage constant;
ω_m: Rotor mechanical angular speed;
θ_e: Rotor electrical position;
F_a, F_b, F_c: Commutation functions of electrical rotor position.

Since the back-EMFs are sinusoidal, the commutation functions F_a, F_b, F_c are also sinusoidal (see Fig. 1), and these 3 functions are 120 °out phase of each other.

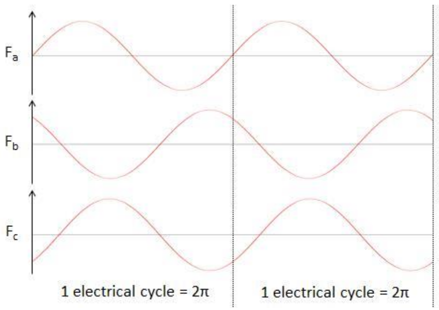


Figure 1. Commutation functions relative to rotor position.

Next, the electrical torque (T_e) is the sum of each phase torque:

$$T_e = T_a + T_b + T_c \quad (3)$$

With K_t is torque constant, each phase torque component T_a, T_b, T_c are defined as:

$$\begin{cases} T_a = K_t i_a F_a \\ T_b = K_t i_b F_b \\ T_c = K_t i_c F_c \end{cases} \quad (4)$$

And the relationship between electrical torque, load torque, and angular speed is indicated in (5):

$$T_e - T_L = J s \omega_m + \beta \omega_m \quad (5)$$

where:
J: motor moment inertia;
β: friction coefficient;
ω_m: rotor angular speed;
T_L: load torque.

And the mechanical angular speed (ω_m) is calculates from the mechanical angular distance (θ_m):

$$\omega_m = s\theta_m \quad (6)$$

where the mechanical angular distance (θ_m) is relative to the electrical angular distance (θ_e) by number pair of poles (PP):

$$\theta_e = PP.\theta_m \quad (7)$$

B. Modeling BLDC Motor on MatLAB Simulink

The detail construction of a BLDC motor is shown in Fig. 2 with the Current I_abc block is built from (1). Similarly, the Torque Te, Omega, Theta_e and Back EMF E_abc blocks are based on (3), (6), (7), and (2) relatively. About the F_abc block, it is based on its definition as mentioned in Fig. 1.

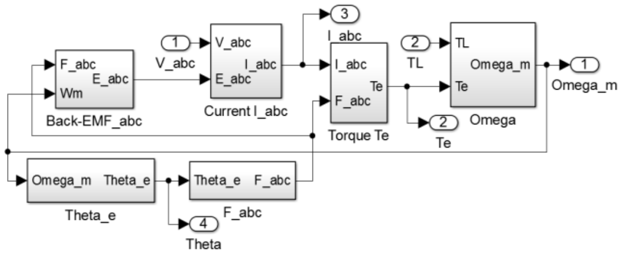


Figure 2. BLDC motor model construction in Simulink.

III. IMC System to Control Speed of BLDC Motor

In Fig. 3, an IMC system which is based on BLDC model is constructed to control the speed of BLDC motor. That means the Forward ($\hat{P}$) and Inverse (Q) are based on BLDC model. In fact, the Forward model is nearly the same as BLDC model except 3 points. First, a subscript M which stands for Model is added to all parameters and coefficients of Forward model. In addition, load torque and friction are neglected (T_{LM}=0, β_M=0). Similarly, the Inverse model also has those 3 differences. Additionally, low-pass filters are added in the Inverse model to reduce the kick-starter effect which is caused from differential components. These low-pass filters have the same coefficients (T_{dM}) as in (8).

$$\frac{d}{dt} = \frac{s}{T_{dM}s + 1} \quad (8)$$

The 3-phase voltage and electrical torque are now become as (9), (10) relatively, and these equations are modelled in Inverse model.

$$\begin{cases} V_a = R_M i_a + \dfrac{1}{T_{dM}s + 1} L_M s i_a + E_a \\ V_b = R_M i_b + \dfrac{1}{T_{dM}s + 1} L_M s i_b + E_b \\ V_c = R_M i_c + \dfrac{1}{T_{dM}s + 1} L_M s i_c + E_c \end{cases} \quad (9)$$

$$T_e = \frac{1}{T_{dM}s + 1} J_M s \omega_m \quad (10)$$

The phase currents are supposed to be sinusoidal ideally with I_0 amplitude as in (11) since back-EMF and 3-phase voltage are also sinusoidal.

$$\begin{cases} i_a = I_o F_a \\ i_b = I_o F_b \\ i_c = I_o F_c \end{cases} \tag{11}$$

Now, combined (11) with (3) and (4) the electrical torque is calculated as (12):

$$T_e = K_t I_o \left[\sin^2(\theta_e) + \sin^2(\theta_e + 2\pi/3) + \sin^2(\theta_e + 4\pi/3) \right]$$

$$T_e = 1.5 K_t I_o \tag{12}$$

Since I_0 can be calculated from the electrical torque T_e, the phase currents now are known as in (11).

A BLDC motor with sinusoidal back-EMF is proved to be equivalent to a DC motor which has 1.5 times of torque constant (K_t) as in (12). This is the principal reason to use DC model-based instead of BLDC's in IMC method, which is clearly illustrated in Section IV.

The general diagram of the IMC system which controls a BLDC speed is presented in Fig. 3. In the Fig. 3, the IMC filter block which controls output response is a low-pass filter as shown in Fig. 4. In this paper, all parameters and coefficients are set as in Table I, these values are copied from [18], [19].

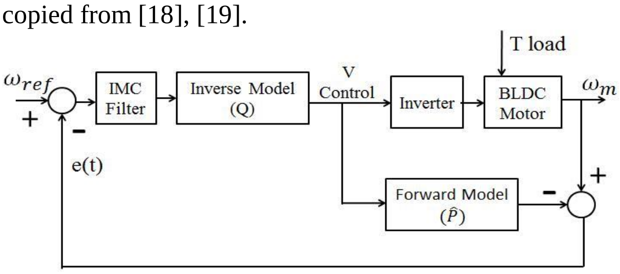


Figure 3. BLDC model-based IMC system for BLDC motor speed control.

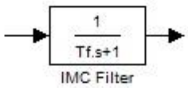


Figure 4. IMC filter block in MatLAB Simulink.

TABLE I. PARAMETERS AND COEFFICIENTS OF MOTOR IN SIMULATION [18], [19]

NO.	Parameters and Coefficients (sign)	Value (unit)
1	IMC filter (T_f)	0.05
2	Differential filter (T_{dM})	0.001
3	Load torque (T_L)	0.03 (Nm)
4	Reference angular speed (ω_{ref})	1400 (RPM)
5	Moment inertia (J)	6.5×10^{-5}($Kg.m^2$)
6	Phase resistance (R)	0.1(Ω)
7	Phase inductance (L)	0.5 (mH)
8	Phase back-EMF constant (K_e)	0.03 (V/(rad/s))
9	Phase torque constant (K_t)	0.03 (Nm/A)
10	DC voltage (V_{DC})	24 (V)
11	Friction (β)	5×10^{-6}(Nm/(rad/s))
12	Simulation time (Tsim)	3 (s)
13	Pairs of pole (PP)	2

IV. GENERAL DC AND STEADY-STATE DC MODEL-BASED IMC SYSTEMS FOR BLDC SPEED CONTROL

The 4 equations (13)-(16) describe the general DC motor, and that number of equations is much less than BLDC's. Since the DC model is used in the Forward and Inverse Model (see Fig. 3), all parameters and coefficients are also with subscript M:

$$V = \frac{di}{dt} L_M + R_M i + E_a \tag{13}$$

with the armature back-EMF E_a:

$$E_a = K_{eM} \omega_m \tag{14}$$

As mentioned in the (12), Section III. The torque is calculated as:

$$T_e = K_{tDC} i = 1.5 K_{tM} i \tag{15}$$

And rotor angular speed (ω_m) is:

$$T_e - T_L = J_M \frac{d\omega_m}{dt} + \beta_M \omega_m \tag{16}$$

At steady-state, all the differential components are neglected (d/dt = 0), so the steady-state DC model is simply described by (17) and (18).

$$V = R_M i + E_a = R_M i + \omega_m K_{eM} \tag{17}$$

$$T_e - T_L = 1.5 K_{tM} i = \omega_m \beta_M \tag{18}$$

Additionally, the load and friction are eliminated (T_L = 0, β_M = 0) in the Forward and Inverse Model. That leads to T_e = 0 and i = 0 according to (18). Combining i = 0 into (17), the relationship between voltage V and angular speed ω_m is simply described by (19):

$$V = E_a = \omega_m K_{eM} \tag{19}$$

With the (19), the Inverse Model (input ω_m, output V) and Forward Model (input V, output ω_m) now become gain functions. That means the IMC controller just includes 2 gain functions and 1 low-pass filter (IMC filter). Obviously, using steady-state DC model to build the Forward and Inverse Model can minimize the amount of calculation and solve the major drawback of IMC method in this particular application.

V. EXPERIMENTS OF BLDC AND STAEDY-STATE DC MODEL-BASED IMC SYSTEMS FOR BLDC SPEED CONTROL

The robustness experiments of 2 IMC systems based on BLDC and steady-state DC model are made under uncertainties of data as in Table II. The Table II shows that the practical parameters and coefficients are assumed to be varied around their model values and reached those limits in the table while the model values are unchanged. In this paper, the system is called robust if its output response can stay within +/- 5% of the reference value during disturbance. In these experiments, the disturbance is the change of load torque as shown in Fig. 5 (upper graph). With the reference angular speed 1400RPM, the output response is shown in Fig. 5 (lower graph). The 2 dash lines in lower graph of Fig. 5 which are above and below the reference value are +/- 5% limits of stable criteria.

TABLE II. PARAMETERS AND COEFFICIENTS RANGES OF ROBUSTNESS EXPERIMENTS

NO.	Parameters and Coefficients (sign)	Upper Limit	Lower Limit	Results
1	Moment inertia (J)	$2J_M$	$0.5J_M$	Fig. 7
2	Phase resistance (R)	$2R_M$	$0.5R_M$	Fig. 8
3	Phase inductance (L)	$1.5L_M$	$0.5L_M$	Fig. 9
4	Phase back-EMF constant (K_e)	$1.2K_{eM}$	$0.8K_{eM}$	Fig. 10
5	Phase torque constant (K_t)	$1.2K_{tM}$	$0.8K_{tM}$	Fig. 10
6	Friction (β)	2β	0.5β	Fig. 11
7	DC voltage (V_{DC})		-20%	Fig. 12
8	Load Torque (T_L)	+20%		Fig. 12
9	All parameters above are at their limits			Fig. 13

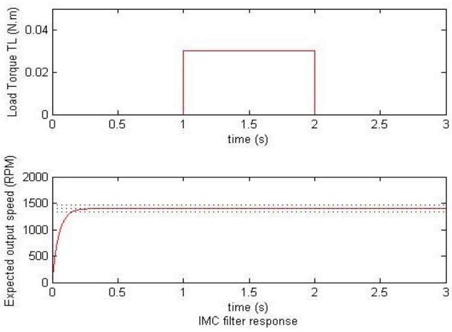
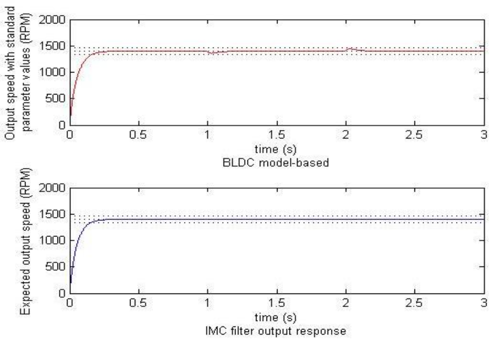
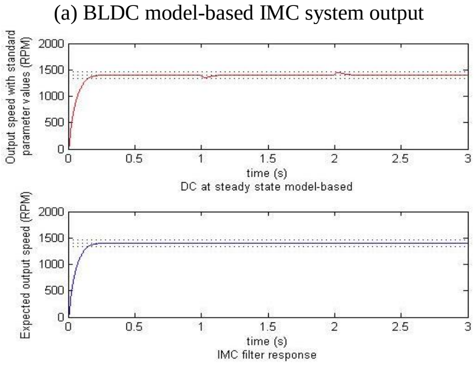


Figure 5. External load disturbance (upper graph) and reference output response (lower graph).



(a) BLDC model-based IMC system output

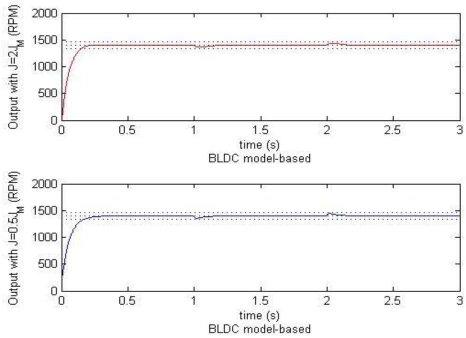


(b) Steady-State DC model-based IMC system output

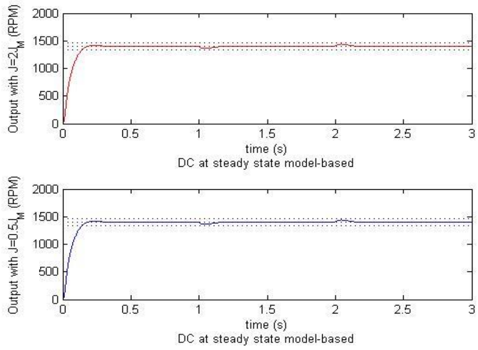
Figure 6. Output response (upper graphs) of 2 model-based IMC systems when practical and model parameters are equal.

When model and practical parameter values are equal, the ideal case is experimented with 2 IMC systems. The results of 2 systems are also compared to IMC filter response in Fig. 6. It is clearly seen that the 2 system outputs response are completely stable within 5% criteria. In Fig. 6, the outputs moreover are nearly the same as the IMC filter response. That shows the system outputs response can be easily controlled by the IMC filter.

Additionally, to test the system's robustness with J, R, L, K_e, K_t, β parameters, the experiences of 2 IMC systems are made when the practical parameter values are considerably different from their model values. Let those parameters be at their limits as mentioned in Table II, the experiment results are shown from Fig. 7 to Fig. 11 respectively. Furthermore, in reality, the DC link voltage of the power inverter and the load torque are usually different from the setting values in the controller. Therefore, the 2 IMC systems are strictly experimented in the worse cases than setting. Those are 20% lower of DC voltage and 20% higher of load. The BLDC motor speed in these 2 situations is shown in Fig. 12.

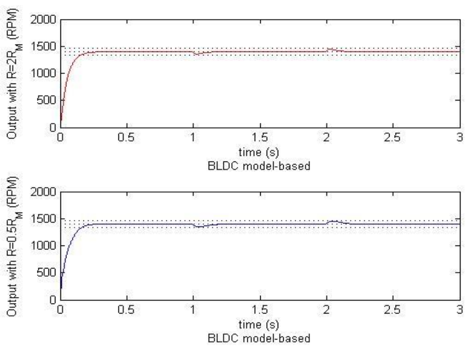


(a) BLDC model-based IMC system output

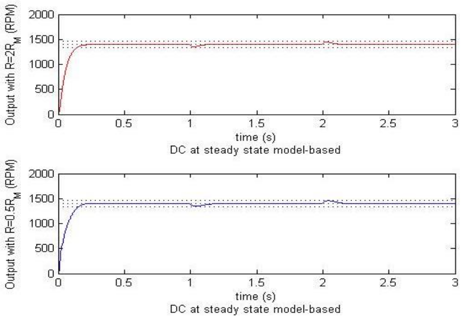


(b) Steady-State DC model-based IMC system output

Figure 7. Output response when J=2J_M and J=0.5J_M.

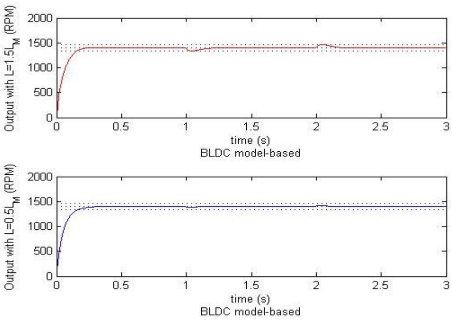


(a) BLDC model-based IMC system output

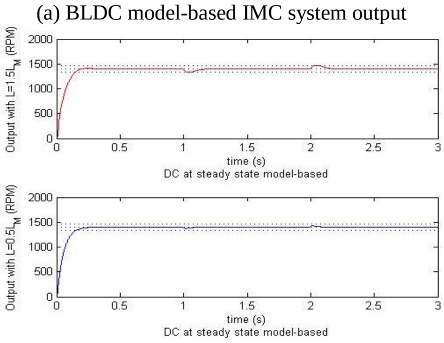


(b) Steady-State DC model-based IMC system output

Figure 8. Output response when R=2R_M and R=0.5R_M.

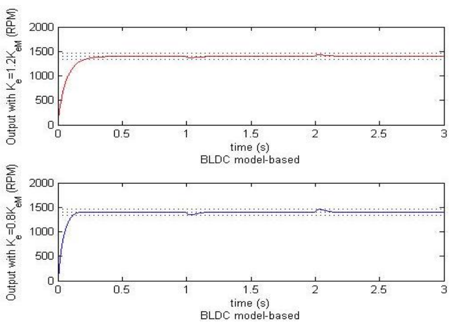


(a) BLDC model-based IMC system output

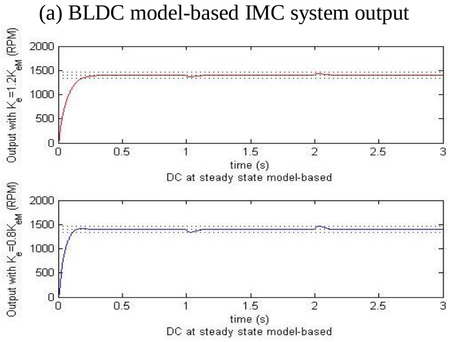


(b) Steady-State DC model-based IMC system output

Figure 9. Output response when L=1.5L_M and L=0.5L_M.

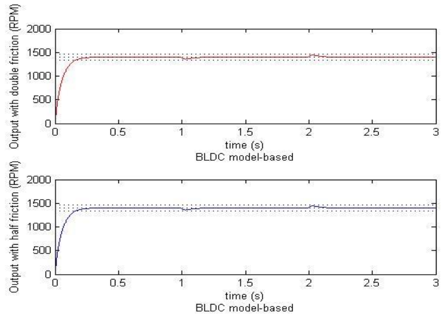


(a) BLDC model-based IMC system output

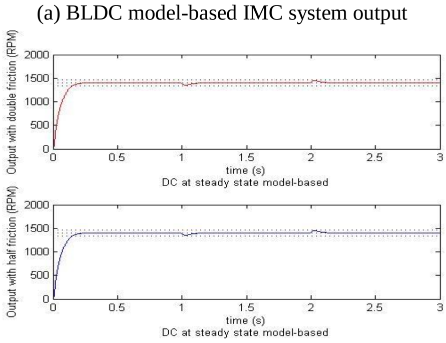


(b) Steady-State DC model-based IMC system output

Figure 10. Output response when K_e=1.2K_{eM}, K_t=1.2K_{tM} and K_e=0.8K_{eM}, K_t=0.8K_{tM}.

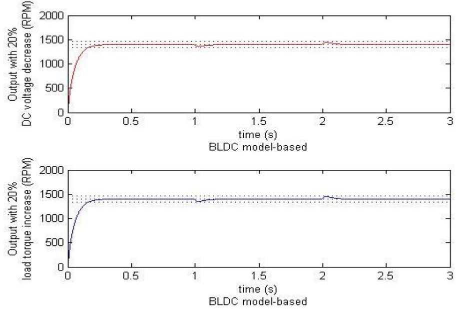


(a) BLDC model-based IMC system output

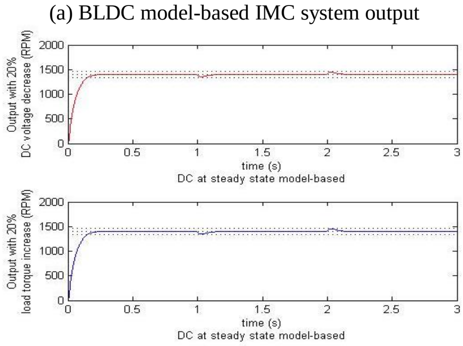


(b) Steady-State DC model-based IMC system output

Figure 11. Output response when double and half friction.

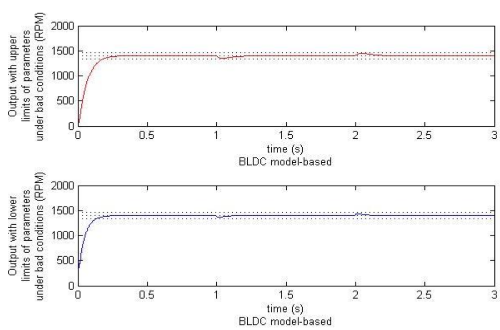


(a) BLDC model-based IMC system output

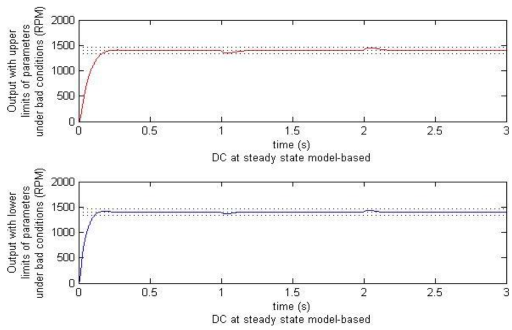


(b) Steady-State DC model-based IMC system output

Figure 12. Output response when 20% DC voltage decrease and 20% load torque increase.



(a) BLDC model-based IMC system output



(b) Steady-State DC model-based IMC system output

Figure 13. Output response when 20% DC voltage decrease, 20% load torque increase and parameters are at upper and lower limits.

To challenge the controller's robustness, the parameters J, R, L, K_e, K_t, β are set at their upper and lower limits when operating in bad conditions as above experiments. The output responses of 2 IMC systems in these ultimate tests are compared in Fig. 13.

Based on experiment results from Fig. 6 to Fig. 13, it is clear that the BLDC angular speed outputs of 2 IMC systems completely stay stable inside the range +/- 5% of reference value during load change in every experiment above. That means they are both well robust with parameters change in wide ranges and under bad working conditions. Similarly, reference [18] proved that 2 IMC systems using BLDC and general DC model for BLDC speed control have the same robustness. Therefore, the IMC system using stead-state DC model also has the same robustness as those twos. However, the steady-state DC model is much simpler than BLDC or general DC model. In other words, the steady-state DC model is better to be used in Forward and Inverse Model of IMC system for BLDC speed control.

VI. Conclusion

Using steady-state DC motor model-based in Forward and Inverse model of IMC system for BLDC speed control, not only all positive features of IMC method are well achieved, but also the calculation is much reduced. In detail, the new IMC system are very robust with a lot of parameters and coefficients (J, R, L, K_e, K_t, β) changing in wide ranges as well as environment conditions (load and DC voltage) being worse. In experiments, the parameters J, R, β can reach to a haft or double of their model values, and the inductance L may be within +/- 50% of its model value. The back-EMF and torque constants can be higher or lower 20% of their model values. Furthermore, some bad operating environments such as 20% DC voltage decrease and 20% load torque increase are also used to test robustness of the new IMC system. The limits of these wide ranges of parameter values are very difficult to happen in practical operating of BLDC motors. Those differences between model and practical parameter values are unavoidable, which may be caused from effect of temperature, electromagnetic field, measurement errors, etc. Beside robustness, the steady-state DC model-based IMC system also achieves other advantages of IMC principle such as easily controlling output response by setting IMC filter coefficient, and simply designing process. Note that, since the steady-state DC model has simpler mathematic model than BLDC and general DC motor model, the designing steps even become easier.

In the near future, an IMC system using steady-state DC model for BLDC speed control will be operated with hardware. Since the amount of calculation is minimized, the hardware may work with an embedded micro-controller such as ARM, PIC.

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