Modeling and Computation of Magnetic Leakage Field in Transformer Using Special Finite Elements

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***Abstract*—Accuracy in theoretical prediction of performance of transformers has become increasingly important to effect economy in design and to ensure reliability of operation. Some of the performance indicators that the power system engineers are concerned with are reactance, electromagnetic forces, short circuit impedance etc. In recent years, finite element methods have been increasingly used. One of the drawbacks in the flux plots so obtained is that at all infinitely permeable iron surface the flux lines are not normal to the iron surface. To eliminate these errors on the boundaries special finite elements i.e. incremental curved elements with linear and cubic variations have been developed. Their incorporation would modify the usual shape functions. With the help of modified shape functions the magnetic vector potentials are solved. Leakage reactance, the magnetic force, energy is calculated and flux plots obtained.**

***Index Terms*—transformer, power system computing finite element method flux plots**

I. INTRODUCTION

Since one of the important features of a transformer is its leakage impedance, improvements in the calculations of this quantity are always searched for. [1]-[3] Analytical methods have been employed in the past for determining the actual flux distribution but most of the analytical methods are not accurate [4], [5]. The drawback of these representations is either the simplified assumptions or the complex nature, which makes the use of it impracticable. Rabin [6] has also presented a solution using Fourier series representations of the ampere-turns distribution, in axial directions only. In the transition region between the two windings, there is an abrupt change in the ampere-turns distribution, and it is difficult to represent this by Fourier series accurately. Numerical modeling techniques are now-a-days well established for transformer analysis and enable representation of all important features of these devices [7]-[9]. In the application of finite element methods for axi- symmetric problems, only the first order and high order triangular elements have been used [10], [11] and flux plots obtained. One of the drawbacks in the flux plots so obtained is that at all infinitely permeable iron surface the flux lines are not normal to the iron surface, i.e. Neumann's boundary condition is not satisfied as it constitutes the natural functional. To eliminate these errors on the boundaries special finite elements i.e. incremental curved elements with linear and cubic variations have been developed.

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II. TRANSFORMER MODELING WITH FINITE ELEMENT METHOD

Mathematical formulation: For the purpose of analysis, the transformer will be assumed rotationally symmetric about a core leg. It will be assumed that all iron is infinitely permeable. In the insulation and winding space the magnetic vector potential must satisfy the vector Poisson's equation. The Poisson's equation in cylindrical coordinates for axi-symmetric problems can be written as:

$$\frac{\partial}{\partial r}\frac{v}{r}\frac{\partial(\phi r)}{\partial r}+\frac{\partial}{\partial z}v\frac{\partial\phi}{\partial z}=J \tag{1}$$

Subject to the boundary conditions: ϕ is specified on the part of the boundary S1 and $\partial\phi/\partial$n on the part of the boundary S2

In the present formulation the given region is divided into a mesh of finite elements, and the vector potential ϕ is approximated in each element. The mesh is a set of rings, each ring having a general curvilinear quadrilateral cross-section that has been revolved around the z-axis. The current density, J is assumed to be directed in the peripheral direction, and thus the vector potential, ϕ is also peripherally directed. Within each element the vector potential ϕ is assumed to vary according to the equation:

$$\phi=\sum N_i\phi_i \tag{2}$$

where N_i is the usual shape functions [10] and ϕ_i is the nodal values of vector potentials.

III. FORMULATION OF INCREMENTAL ELEMENTS

Fig. 1 (a) and Fig. 1(b) shows para- linear and parabolic incremental elements respectively where EF is the boundary on which the condition $\partial\phi/\partial n=\alpha(x,y)$ holds.

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One side of the element, represented by the nodes 4, 5 and 6 is lying on the boundary EF. The opposite side, having nodes 1, 2 and 3 on it, is lying on the boundary line GH, which is very close to the boundary line EF. The variations in the η and ξ directions are linear and parabolic respectively.

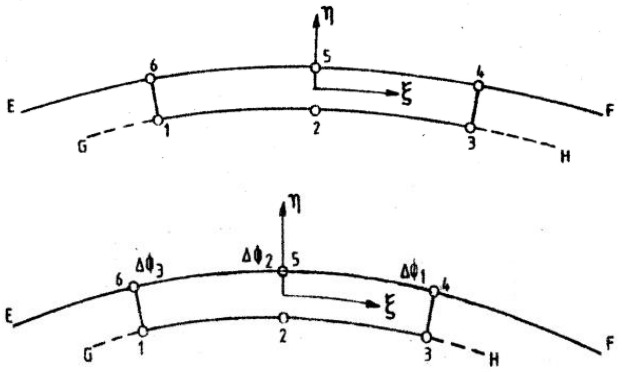

Figure 1. (a) Para-Linear incremental element (b) parabolic incremental element

Higher or lower variation in ξ direction is possible. Thus basically an incremental element is either a linear-linear, para-linear or a cubi- linear element. The shape functions for the curved para-linear element can be written as:

$$N_1 = n_1\eta_1,\ N_2 = n_2\eta_1,\ N_3 = n_3\eta_1,\ N_4 = n_3\eta_2,$$
$$N_5 = n_2\eta_2,\ N_6 = n_1\eta_2 \tag{3}$$

where $n_1 = 0.5[\xi(\xi-1)]$, $n_2 = (1-\xi^2)$, $n_3 = 0.5[\xi(\xi+1)]$, $\eta_1 = 0.5(1-\eta)$, and $\eta_2 = 0.5(1+\eta)$

Assuming,

$$\Delta\phi_1 = \phi_4 - \phi_3,\ \Delta\phi_2 = \phi_5 - \phi_2 \text{ and } \Delta\phi_3 = \phi_6 - \phi_1$$

The usual nodal potential values, $\{\phi\}^e$ and the incremental nodal potentials, $\{\bar{\phi}\}^e$ are related by:

$$\{\phi\}^e = [C]\{\bar{\phi}\}^e \tag{4}$$

where $[C] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ and

$$\{\bar{\phi}\}^e = [\phi_1, \phi_2, \phi_3, \phi, \Delta\phi_1, \Delta\phi_2, \Delta\phi_3]^T$$

Matrix [C] is the required connection matrix. The element in terms of the incremental nodal potentials is represented in Fig. 1(b). The potential at any point in para-linear element is given by:

$$\{\phi\} = [N]\{\phi\}^e$$

This expression can be modified, with the help of Eq.(4) to connect the potential, ϕ, with the incremental parameters. Thus:

$$\phi = [N][C]\{\bar{\phi}\}^e$$
$$= [\bar{N}]\{\bar{\phi}\}^e \tag{5}$$

where $[\bar{N}] = [N][C] = [N_1 + N_6, N_2 + N_5, N_3 + N_4, N_4, N_5, N_6]$

Let:

$$[\bar{N}] = [N_1', N_2', N_3', N_4, N_5, N_6,] \tag{6}$$

where N_1', N_2', and N_3' are the new shape functions at nodes 1, 2 and 3 respectively, and can be shown to be equal to:

$$N_1' = n_1,\ N_2' = n_2,\ N_3' = n_3 \tag{7}$$

For the calculation of gradient matrix the Jacobin matrix and its determinant and the original shape functions of the para- linear element as given by (3) are required. Thus the modified shape functions, (6) and the original shape functions, (3) will have to be stored. An alternative approach is to calculate the Jacobian matrix and its determinant by using the modified shape functions. For this the coordinates of the nodes will have to be modified, since it is convenient to modify the coordinates of the nodes instead of retaining the original shape functions. This latter approach is used here. Using the modified shape functions the x and y coordinates can be written as:

$$x = [N]\{x\}^e = [\bar{N}][\bar{x}]^e$$
$$y = [N]\{y\}^e = [\bar{N}][\bar{y}]^e \tag{8}$$

where:

$$[\bar{x}]^e = [C]^{-1}\{x\}^e = [x_1, x_2, x_3, \Delta x_1, \Delta x_2, \Delta x_3,]$$
$$[\bar{y}]^e = [C]^{-1}\{y\}^e = [y_1, y_2, y_3, \Delta y_1, \Delta y_2, \Delta y_3,]$$

and $\Delta x_1 = x_4 - x_3$, $\Delta x_2 = x_5 - x_2$, $\Delta x_3 = x_6 - x_1$, and so on. $[\bar{x}]$ and $[\bar{y}]$ are the modified coordinates of the element. Only the coordinates of the nodes lying on the boundary EF needs modifications. With the modified shape functions and coordinates the procedure of calculating the element properties by numerical integration will remain unaltered. The elements fulfill the necessary conditions of convergence. The compatibility conditions are also satisfied. The incremental elements must satisfy the Neumann type of boundary condition $\partial\phi/\partial n = \alpha(x, y)$. For this purpose α will appear in the final potential vector of the global equations as the known values of the nodal potentials for the nodes on the boundary EF. Using modified shape functions reluctance matrix, [R] and current load vector {I} is carried out numerically.

IV. Reactance Calculation

The leakage reactance can be calculated as pointed out by Anderson [11] and Silvester [12], by calculating the stored energy in the leakage field. The leakage reactance X, using the stored energy W, is given by:

$$X = 2\omega W / I^2$$

where:

$$W = 0.5\int\int_v J.\phi dv \tag{9}$$

ω is supply frequency, and I is the current peak or effective, for which the energy W was evaluated. For the axi-symmetric problems, the radial and axial components of the short circuit electromagnetic force can be written as:

$$\begin{pmatrix} F_r \\ F_z \end{pmatrix} = \int_v J \begin{pmatrix} B_z \\ -B_r \end{pmatrix} dv \tag{10}$$

where B_z and B_r are the flux densities in axial and radial directions respectively.

TABLE I. Computed Value of Leakage Reactance

Sr. No.	No. of Nodes	Elements Type	No. of Elements	Reactance p.u.	No. of integrating points per Element
1.	100	L. Q.	81	0.56046	4
2.	96	P. Q.	25	0.58683	9
3.	100	C. Q.	15	0.58620	16
4.	180	L. Q.	154	0.56815	4
5.	176	P. Q.	49	0.59315	9
6.	184	C. Q.	30	0.59220	16
7.	96	P. Q.	25	0.59167	4
8.	176	P. Q.	49	0.59380	4
9.	96	P. Q.	25	0.57480	9
	44*	P. Q.	20		4
10.	96	P. Q.	25	0.57668	4
	44*	P. Q.	20		4
(* Incremental)					
Tested value of reactance = 0.5870 p.u.					
Rabin's calculated value = 0.5829 p.u.					
Anderson's calculated value using 3496 first order triangular elements 1833 nodes = 0.5886 p.u.					

V. Sample Problem

Andersen [10] and Silvester [11] have used the Rabins case 2 to illustrate their finite element analysis. This case is taken to illustrate the various aspects of the present finite element analysis. In the analysis numerically integrated general quadrilateral iso-parametric elements with or without curved sides have been used. Transformer leakage field analysis is dealt in the present discussion.

The element having linear, parabolic and cubic interpolation functions and belonging to serendipity family have been used. These elements are shown in Fig. 1(a) and (b). For comparison, the solution domain was divided into elements so that the total number of nodes are approximately same for various type of elements. The results obtained for approximately 100 nodes and 180 nodes are tabulated in Table I.

The agreement with the tested result for reactance is very good. The flux plots for 180 nodes case are shown in Fig. 3. The agreement with Anderson [10] and Silvester [11] is good. The flux plots for 100 nodes case are similar. When the number of integrating points for linear, parabolic and cubic elements are 4, 9 and 16 respectively, the present study indicates an approximate time ratio 1:5:15 for the formation of reluctance matrix of an element and a total problem time ratio 3:4:7, on the basis of equal number of nodes for an analysis by linear, parabolic and cubic elements. However, if the numbers of integrating points in each parabolic element are chosen to be 4, then a time saving of approximately 30% in the formation of reluctance matrix, is achieved and the total problem time is of the same order as for linear elements for a problem having same number of nodes.

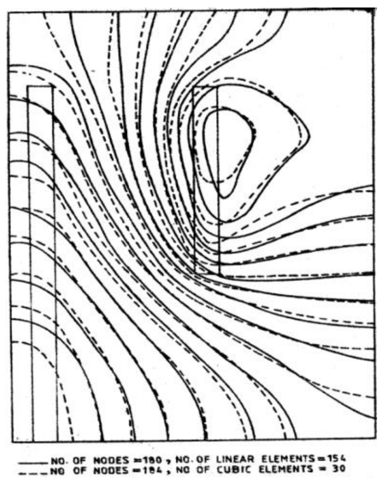

Figure 2. Leakage field of the transformer using linear and cubic elements

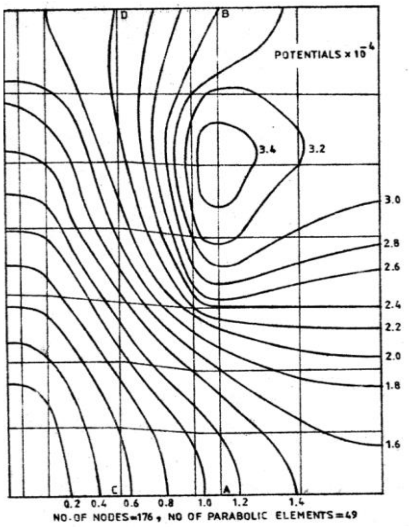

Figure. 3. Leakage field of the transformer using parabolic elements

The problem was also analyzed by the use of incremental elements. The 96 Node, 25 parabolic elements Fig. 2 and Fig. 3, mesh was used by applying the incremental elements around it. The efficiency and computational effort are of the same order as that when no incremental elements are used. The use of incremental elements has given a reactance value, hence the stored energy is also slightly less than the tested value. This is due to the fact that the incremental elements allow the flux density variation in only one direction and this effect is also carried to some extent to the normal element on the side of which the incremental element is attached.

In case of normal elements the solution has been possible by assuming a known potential at one point in the domain. In present work zero potential is assumed at lower left corner. No such assumption is necessary, when field is analyzed with the help of incremental elements

VI. Conclusion

The use of incremental numerically integrated higher order isoparametric elements offers many significant advantages. These are easy to use, boundary conditions and curvatures are properly accounted for; they yield fast and reliable solutions. On the basis of improved accuracy, and computational efforts required to solve a given problem and ease in input data preparation, the parabolic elements are preferred for routine work. In addition, total computation time of the order of linear elements is obtained if parabolic elements with four integrating points are used. The use of cubic elements is recommended only in cases where flux concentration is high and the geometry permits the use of only few elements.

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