Joint Range and Angle Estimation Based on Sub-Array Scheme with Frequency Diverse Array Radar

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Abstract—Frequency Diverse Array (FDA) has its unique advantage in realizing Low Probability of Intercept (LPI) technology for its beam pattern is related to the range, time and angle. In this paper, a sub-array scheme of range-angle joint estimation of a target for Frequency Diverse Array (FDA) radar is proposed. To solve the problem of distance and angle response coupling of basic ULA FDA radar, the entire array is divided into two sub-arrays, which employs two different frequency offsets. For aperture extension, each sub-array adopts difference co-array structure to provide $O(N^2)$ degrees of freedom by only $N$ physical sensors when the second-order statistics of the received data is used. Therefore, the targets range and angle can be estimated directly with the subspace-based multiple signal classification algorithms. The estimation performance is examined by analyzing the Cramer-Rao Lower Bound (CRLB) versus Signal-to-Noise Ratio (SNR). Simulation results show that the proposed approach is effective to realize the joint range and angle estimation of FDA.

Index Terms—Frequency Diverse Array (FDA), range-angle estimation, difference co-array, Cramer-Rao Lower Bound (CRLB)

I. INTRODUCTION

Low Probability of Intercept (LPI) is an effective means of modern battlefield against a variety of electronic surveillance and interference measures. In modern radar systems, many features are combined together to improve LPI performance of radar. These features focus on performance in the transmitted waveform of the transmitter, beam pattern of the antenna and the beam scanning pattern. To achieve the low interception performance of radar with the aid of the advanced beam scanning mode, and by virtue of the inherent advantages of its antenna pattern has become a new research direction.

Because of its unique advantages such as non-inertial beam scanning, easy and fast beam scheduling and energy management, the phased array radar has been widely used. However, the beam steering of the conventional phase-array radar is fixed in an angle for all the ranges [1], [2]. If we want to focus the antenna beams in the directions with different ranges, multiple antennas or a multi-beam antenna must be required. To resolve this problem, distributed Multiple-Input Multiple-Output (MIMO) radar is often suggested [3], but time and phase synchronization becomes a new technical challenge [4]-[7]. Frequency Diverse Array (FDA), also known as frequency re-use radar, it’s most important difference from a conventional phased-array is that a uniform inter-element frequency offset is applied across the aperture of uniform linear array of antennas, and the use of frequency increment can generate an beam pattern that is a function of range, time, and phase [8]. While the beam pattern is coupled in range and angle, the target’s range and angle cannot be estimated directly by the FDA radar. In reference [9], a Uniform Linear Array (ULA) double-pulse Frequency Diverse Array (FDA) radar scheme was been proposed to estimate the range and angle of targets. However, the range and angle information is not estimated simultaneously.

In this paper, we propose a sub-array scheme on the FDA radar, with the aim to locate the target in range-angle domain. The idea is that the whole FDA linear array is divided into two sub-arrays which employ two different frequency shifts. Since the sub-arrays offers decoupled range and angle response, the range and angle information of targets can be estimated with the subspace-based multiple signal classification algorithms. The sub-array is equivalent to two sets of equations to solve two unknown quantities, and the closed solution of the unknown quantity can be directly determined.

II. FDA RADAR

A Uniform Linear Array (ULA) FDA radar, which is illustrated in Fig. 1, and assume that the waveforms radiated from each antenna element are identical with a frequency increment of $\Delta f$ Hz applied across the elements. That is, the radiation frequency of the $m$ th element is:

$$f_m = f_0 + (m-1) \cdot \Delta f, m = 0, 1, \cdots, M - 1$$ (1)
where, \( f_0 \) is the radar operating carrier frequency and \( m \) is the number of the array elements.

\[
\mathbf{a}(0, r) = [1 e^{-j2\pi f_0 d \sin \theta / c_0}, e^{-j2\pi (M-1) d \sin \theta / c_0}, \ldots e^{-j}]^T
\]

where \( \theta \) is the azimuth direction and \( r \) is the slant range, \( d \) is the element spacing, \( c_0 \) is the speed of light and \( T \) is the transpose. Since \( f_0 \gg \Delta f \) and \( r \gg (M-1)d \sin \theta \), in an amplitude sense the approximation is taken by ignoring the nonlinear phase term \( 2\pi \cdot (m-1)^2 \Delta f \cdot d \sin \theta / c_0 \), which has ignorable impacts [11]. This approximation is suitable for the difference co-array processing mentioned later.

Figure 1. Uniform linear FDA with identical frequency increment

The use of frequency increment generates an antenna pattern that is a function of range, time, and phase. The FDA radar can offer a range-angle dependent beam-pattern, which is of importance as this will cause that targets can be located in the range-angle domain. However, the basic FDA radar cannot estimate directly the range and angle of a target due to the couple of range and angle dimensions.

III. THE PROPOSED RANGE-ANGLE LOCALIZATION METHOD

To decouple the range and angle peaks and estimate both the range and angle of targets, we propose a sub-array scheme on the FDA. We divide the whole array into two sub-arrays, each sub-array has \( M \) elements and uses different frequency increment, which is illustrated in Fig. 2. The frequency increment are \( \Delta f_1 \) and \( \Delta f_2 \). Subarrays1 is composed of \( M \) omnidirectional sensors whose locations are non-uniformly arranged as \([0, d_1, \ldots, d_{M-1}]\), where \( d_{M-1} = M (\lambda / 2) \) denotes the distance between the first sensor (reference sensor) and the \( m \)th sensor and is an integral multiple of half a wavelength. Subarray2 is located at \([D, d_1, D + d_1, \ldots, d_{M-1} + D] \), which is identical with a shifted array of subarray1, and the shifting distance is \( D \).

Figure 2. FDA radar with two sub-arrays

Taking the first element as the reference for the array, the steering vector is given by (2) [10]:

\[
\mathbf{a}(0, r) = [1 e^{-j2\pi f_0 (M-1) d \sin \theta / c_0}, e^{-j2\pi (M-1) \Delta f \sin \theta / c_0}, \ldots e^{-j}]^T
\]

The difference co-array of the first sparse array (subarray 1) is determined by the set of position differences between sensor elements.

\[
\Omega = \{(d_m - d_{m+1})|m_0 = 0, m_2 = 0\}
\]

The general form of the output model for each sub-array is:

\[
x_1(t) = A_1 s(t) + n_1(t) \quad t = 1, 2, \ldots, T
\]

\[
x_2(t) = A_2 s(t) + n_2(t) \quad t = 1, 2, \ldots, T
\]

where \( x_1(t), x_2(t), s(t), n_1(t), n_2(t) \) denote the subarray 1 output vector, subarray 2 output vector, incident source vector, and additive noise vector of the \( t \)th snapshot, respectively, and \( T \) is the number of snapshots. The manifold matrix of subarray 1 \( A_1 \) consists of \( K \) steering vectors.

\[
A_1 = \{a(\theta_1, r_1), a(\theta_2, r_2), \ldots, a(\theta_K, r_K)\}
\]

where the \( \theta_k \) and \( r_k \), \( k = 1, 2, \ldots, K \) are respectively the direction of arrival and slant range of the \( k \)th target, and the steering vector \( \mathbf{a}(0, r) \) is:

\[
\mathbf{a}(0, r) = [1, \nu(d_1, \theta, r), \ldots, \nu(d_{M-1}, \theta, r)]^T
\]

with:

\[
\nu(d_m, \theta, r) = \exp[-j(2\pi f_0 d_m \sin \theta / c_0 - 2\pi \Delta f_m r / c_0)]
\]

Benefiting from the un-correlation among sources and noisy, the covariance matrix of the array output vector in (4) can be formulated as:

\[
\mathbf{R}_x = \mathbb{E}

\left( x(t) x^H(t) \right) = A_1 \mathbf{R}_x A_1^H + \sigma_n^2 \mathbf{I}
\]

\[
= A_1 \begin{pmatrix}
\sigma_1^2 & \sigma_2^2 & \ldots & \sigma_K^2
\end{pmatrix}

A_1^H + \sigma_n^2 \mathbf{I}
\]

where, \( \mathbf{I} \) is an identity matrix, \( \sigma_1^2, \sigma_2^2, \ldots, \sigma_K^2 \) are the variances of the \( K \) sources, and \( \sigma_n^2 \) is the variance of Independently Identically Distributed (IID) noise. A new vector \( \mathbf{y} \) is obtained:

\[
\mathbf{y} = \nu(\mathbf{R}_x) = \nu(\mathbf{A}_1 \mathbf{R}_x \mathbf{A}_1^H) + \sigma_n^2 \nu(\mathbf{I}) = (\mathbf{A}_1 \otimes \mathbf{A}_1) \mathbf{p} + \sigma_n^2 \mathbf{I}_M
\]

Define the virtual array manifold matrix \( \mathbf{B} = (\mathbf{A}_1 \otimes \mathbf{A}_1) \) consists of \( K \) virtual steering vectors:

\[
\mathbf{B} = [a'(\theta_1, r_1) \otimes a(\theta_1, r_1), \ldots, a'(\theta_K, r_K) \otimes a(\theta_K, r_K)]
\]

Select the distinct entries of \( \mathbf{y} \) and from a new data of vector \( \hat{\mathbf{y}} \), and then the model with a new virtual manifold \( \hat{\mathbf{B}} \) can be represented as:
\[ \hat{y} = \hat{B}p + \sigma^2 \hat{e} \]

where, the virtual array manifold \( \hat{B} = \{\hat{a}(\theta_1, r_1), \hat{a}(\theta_2, r_2), \ldots, \hat{a}(\theta_k, r_k)\} \), \( \hat{e} \) has \( 2M_a + 1 \) entry vector with all the entries being zero except the \( M_a + 1 \) th one being 1, and the steering vector is:

\[
\Phi = \begin{pmatrix}
  e^{j(2\pi f_d/2\sin\theta_1/c_0 - 2\pi f_d/c_0)} \\
  e^{j(2\pi f_d/2\sin\theta_2/c_0 - 2\pi f_d/c_0)} \\
  \vdots \\
  e^{j(2\pi f_d/2\sin\theta_k/c_0 - 2\pi f_d/c_0)}
\end{pmatrix}
\]

where, define:

\[
R_{\text{average}} = \frac{1}{(M_a + 1)} \sum_{i=1}^{M_a + 1} R_i
\]

Finally, the proposed range-angle localization method can be summarized as follows:

1) Compute the local covariance estimates \( R_{\alpha} \) of each subarray using (9). A total of \( M^2 \) second-order statistics values are computed which are arranged in the form of the vector \( y \in \mathbb{C}^{M^2} \).

2) Compute the difference set of the physical array. Let \( 2M_a + 1 \) denote the number of elements in its ULA segment (always odd), extending from \(-M_a\) to \( M_a\). Extract the rows of \( y \), which correspond to the elements in the ULA segment of the difference co-array, to obtain the vector \( \hat{y} \in \mathbb{C}^{2M_a + 1} \). Order the element in \( \hat{y} \) such that the \( i \) th row corresponds to the element in the ULA segment at location \(-M_a + i - 1\).

3) Apply spatial smoothing technique on \( \hat{y} \), and construct the positive semi-definite matrix according to (12).

4) Apply subspace based algorithms (Root-MUSIC and TLS-ESPRIT) on \( R_{\text{average}} \) [12], [13], and find the range and angle estimates using (15).

IV. CRLB DERIVATION

In this section, the Cramer-Rao lower bound estimation performance versus Signal-to-Noise Ratio (SNR) is derived. We suppose the observed array output sample is circularly Gaussian distributed with the following mean \( \mu = [1, \ldots, e^{-j(M_a-1)\theta_1}, \ldots, e^{-j(M_a-1)\theta_k}]^T \) and covariance \( \Gamma = 1/\sqrt{\text{SNR}} \cdot I \), where \( \theta_i = 2\pi f_d \sin\theta_i - \Delta f \lambda_i/c_0 \), \( i = 1, 2 \).

Under the Deterministic Assumption [14], the parameter vector to be estimated is given by \( \eta = [\theta_i, r_i] \) [1]. The Fisher Information Matrix (FIM) can then be derived as:

\[
\mathbf{J} = 2\text{Re} \left[ \frac{\partial \mathbf{u}}{\partial \eta} \Gamma^{-1} \frac{\partial \mathbf{u}^T}{\partial \eta} \right] = 2\text{SNR} \sum_{m=0}^{M-1} \left[ \begin{array}{cccc}
2k_1^2 & k_1k_2 & k_1k_3 \\
k_1k_2 & k_2^2 & k_2k_3 \\
k_1k_3 & k_2k_3 & k_3^2
\end{array} \right]
\]

\[
\hat{a}(\theta, r) = [\nu(-M_a\lambda/2, \theta, r), \ldots, 1, \nu(M_a\lambda/2, \theta, r)]^T
\]

Then, the technique of spatial smoothing can be applied for the coherent signal model given by (12). It is easy to check that \( \hat{y}_1 \) and \( \Phi \) are:

\[
\hat{y}_1 = \Lambda \Phi^T \mu + \sigma^2 \hat{e}
\]

The CRLB for the estimates of the target angle and range are the first two diagonal elements of the inverse of the FIM.

\[
\text{CRLB}_{\theta} = \frac{1}{K} \left[ \mathbf{J}^{-1} \right]_{1,1} = \frac{k_1^2 + k_2^2}{2 \cdot \text{SNR} \cdot k_2^2 \sum_{m=0}^{M-1} m^2}
\]

\[
\text{CRLB}_{r} = \frac{1}{K} \left[ \mathbf{J}^{-1} \right]_{2,2} = \frac{1}{\text{SNR} \cdot (k_2 - k_1)^2 \sum_{m=0}^{M-1} m^2}
\]

The total estimation variance can then be evaluated by:

\[
\sigma^2 = \sqrt{\text{CRLB}_{\theta} + \text{CRLB}_{r}}
\]

V. SIMULATION RESULTS AND DISCUSSIONS

Fig. 3 shows the parameter estimation of the basic ULA FDA radar and the proposed sub-array FDA radar. For the source at \( (20^\circ, 9.6\text{km}) \), Fig. 3(a) shows that the parameter estimation of the basic ULA FDA radar is a straight line. It means the beam-pattern is coupled in range and angle, the target’s range and angle cannot be estimated directly by the basic ULA radar. Fig. 3(b) shows that the parameter estimation of the proposed sub-array FDA radar is a point. It means proposed sub-array FDA radar can estimate both the angle and the distance to target, which can locate the target directly. Therefore, from a functional point of view, the proposed sub-array FDA radar is far better than the basic ULA FDA radar.
Fig. 4 and Fig. 5 show, respectively, the comparative CRLB on target angle and range estimates versus SNR, averaged over 1000 Monte Carlo simulations, for $T=800$ snapshots, the source locate at $(10^\circ, 9\text{km})$. It can be seen that the sub-array FDA radar approach gives a satisfactory estimation performance. Also, the TLS-ESPRIT method performs better than the Root-MUSIC. The actual estimation performance can achieve the theoretical CRLB when the SNR is better than 15dB.

VI. CONCLUSION

FDA provides a range-angle beam-pattern, but it cannot estimate directly both the range and angle of a target based on the standard ULA FDA. In this paper, we devise a sub-array scheme on FDA radar for range and angle estimation. The essence is to divide the FDA elements into two sub-arrays, which offer decoupled range and angle responses and target positions can be estimated by the subspace-based algorithm (Root-MUSIC and TLS-ESPRIT). In order to aperture extension, difference co-array is employed in each sub-arrays, and more targets can be distinguished than the physical sensors. The numerical simulation results show that a satisfactory estimation performance can be got. The derived CRLB also can be used to optimally design the sub-array frequency shifts.

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