Comparison of Data Fusion Techniques for Human Knee Joint Range-of-Motion Measurement Using Inertial Sensors

Olubiyo O. Akintade and Lawrence O. Kehinde  
Department of Electronic and Electrical Engineering, Obafemi Awolowo University, Ile-Ife, Nigeria  
Email: {oakintade, lkehinde}@oauife.edu.ng

Abstract—This work uses Kalman and complementary filters to integrate data from inertial sensors (accelerometer and gyroscope) for the purpose of tracking human knee joint movement and then compares results from these filters. A combination of a tri-axial accelerometer and a tri-axial gyroscope (MPU-9150) was used as sensor, mbed NXP LPC1768 microcontroller and XBee wireless communication modules were also used. Both filters show high repeatability (<1.0°), indicating usability. Though the Kalman filter is the most appropriate (theoretically) for data fusion of noisy measurements, it is computationally intensive. Results in this work however show that the complementary filter (which is cheaper and much simpler to implement) works equally well for this application.

Index Terms—accelerometer, gyroscope, knee joint, complementary filter, Kalman filter, range-of-motion

I. INTRODUCTION

The question of quantitative assessment of progress in stroke rehabilitation patients in developing countries prompted this study. Patients go into rehabilitation with the aim of regaining functional use of their limbs. This is usually a very rigorous process and can take a very long time (and at times functional use of the limb is not fully regained). One way to quantify progress in rehabilitation patients is to monitor joint movements by measuring joint angles. Human joint angles are traditionally measured with a goniometer [1], which is not capable of continuous angle measurements. Continuously monitoring human physiological parameters within and outside the hospital environment is however necessary in most applications. Thus, this work develops an affordable electronic goniometer that is capable of continuously tracking the human knee joint angles using inertial sensors (accelerometer and gyroscope).

Inertial sensors are fast becoming ubiquitous because of their usefulness in numerous applications. They are used, for example, in smart phones to track orientation. An accelerometer is used to measure proper acceleration, which is the acceleration relative to a free-fall while a gyroscope measures rate of change of angle with time. Though any one of these sensors can be used to track angle [2], one can integrate the accelerometer and gyroscope angle data for better angle estimates. This paper describes and compares two data fusion techniques, namely, Kalman filter and complementary filter. MPU-9150 [3, 4] inertial measurement unit (IMU) which combines accelerometer and gyroscope in a single unit is used as sensor, ZigBee protocol is used for wireless communication and mbed NXP LPC1768 as microcontroller.

II. RELATED WORKS

Most studies on wearable sensor systems combine accelerometers and gyroscopes for tracking orientation of body segments. Reference [5] integrated data from both accelerometer and gyroscope for better angle estimates of the shoulder and elbow joints. Reference [6] monitored knee joint angle by using a combination of MARG sensor (Magnetic, Angular Rate, Gravity) and pressure sensor. Although the accuracy of their system is high, the MARG sensors are expensive and may not be appropriate for low cost applications. Reference [1] compared a number of techniques used in monitoring human joint angle. These techniques include the use of conductive fiber optic sensors, flex sensors and IMU. The study reports that the accuracy of the IMU monitoring system is higher than the others. Reference [7] showed that orientation obtained by integrating gyroscope rotational rate can be improved by fusion with inclination information obtained from accelerometers. The difference between gyroscope and accelerometer tilt was used as an input to a Kalman filter to obtain a more accurate tilt angle. Reference [8] used only gyroscopes to wirelessly monitor human knee joint angles.

In a recent work, [9] designed an inertial joint angle tracker by placing IMU on each body segment. The tracker integrates data from accelerometers and gyroscopes using the unscented Kalman filter. Reference [2] used accelerometers and gyroscopes independently to track human knee joint angles before comparing the data obtained from the two sensors. The work concluded that though any one of these inertial sensors can be used to track angle independently, accelerometers are good for tilt (static) angle measurement while the gyroscope is good for range-of-motion (dynamic) measurement.

The goal of this study is to demonstrate the feasibility of integrating accelerometer and gyroscope angle data in
a single system using data fusion techniques, that is, complementary filter and the computationally intensive Kalman filter. Then, the outputs of these filters will be compared with the aim of facilitating low cost quantitative assessment of pathological gait.

III. ANGLE MEASUREMENT USING INERTIAL SENSORS

A hybrid (combination of wireless and wired communications) knee joint angle measurement system comprising of two MPU-9150 as sensors, an mbed microcontroller and two XBee transceivers (for wireless communication) is developed. The block diagram is shown in Fig. 1. Sensors 1 and 2 are placed on the thigh and shank, respectively.

The problem with accelerometer is that it does not only measure gravitational force but also force due to vibration. In other words it is sensitive to all forces acting on it and not only the gravitational force [10]. Therefore all the other forces acting on the accelerometer except that due to gravity introduce noise into our measurement as shown in Fig. 3.

B. Angle Measurement with Gyroscope

The gyroscope measures the rate of change of angle with time (therefore it should output zero when it is stationary). If \( G(t) \) is the output of the gyroscope at time \( t \), then:

\[
G(t) = \frac{d\theta(t)}{dt} = \dot{\theta}(t)
\]

Therefore,

\[
\theta(t) = \int_0^t \dot{\theta}(\tau) d\tau
\]

Or,

\[
\theta(k) \approx \sum_{k=0}^{n} \dot{\theta}(k)T_k
\]

A. Angle Measurement with Accelerometer

The accelerometer is used to measure force due to gravity. The angle of inclination the accelerometer makes with the three axes is determined using Fig. 2. \( \theta_x \), \( \theta_y \) and \( \theta_z \) are the angles between the accelerometer and the \( x \)-, \( y \)- and \( z \)-axes, respectively. The components of the gravitational force, \( F \), acting on the accelerometer along the \( x \)-, \( y \)- and \( z \)-axes are \( F_x \), \( F_y \) and \( F_z \), respectively. If the accelerometer’s outputs on the \( x \)-, \( y \)- and \( z \)-axes are \( A_x \), \( A_y \) and \( A_z \), respectively and \( A_{\text{anomaly}} \) is the sensitivity of the accelerometer, then

\[
\begin{align*}
F_x &= \frac{A_x}{A_{\text{anomaly}}} \\
F_y &= \frac{A_y}{A_{\text{anomaly}}} \\
F_z &= \frac{A_z}{A_{\text{anomaly}}} \\
F &= \sqrt{F_x^2 + F_y^2 + F_z^2}
\end{align*}
\]

Therefore, the accelerometer angles in Fig. 2 are

\[
\begin{align*}
\theta_x &= \cos^{-1}\left(\frac{F_x}{F}\right) \\
\theta_y &= \cos^{-1}\left(\frac{F_y}{F}\right) \\
\theta_z &= \cos^{-1}\left(\frac{F_z}{F}\right)
\end{align*}
\]
is not zero when the and the accelerometer is the previous angular rate obtained from the (9).

Consequently, the gyroscope will drift if with a gyroscope (as shown in Fig. 4). The output from the gyroscope is stationary. Reference [2] showed the problem with tracking angle measurement and it does not drift while the gyroscope is stationary.

The accelerometer is good for low frequency angle measurement and it is not susceptible to external forces. The complementary filter leverages on the advantages of each sensor by passing angles obtained from the accelerometer and gyroscope through a digital low-pass and a digital high-pass filter respectively. The complementary filter is excellent for high frequency angle measurement and it does not drift while the gyroscope is stationary.

A. Complementary Filter

The accelerometer is good for low frequency angle measurement and it does not drift while the gyroscope is excellent for high frequency angle measurement and it is not susceptible to external forces. The complementary filter leverages on the advantages of each sensor by passing angles obtained from the accelerometer and gyroscope through a digital low-pass and a digital high-pass filter respectively. The complementary filter is shown in Fig. 5.

Equation (9) is the mathematical implementation of the complementary filter.

IV. DATA FUSION

In this work, data fusion is used to describe the integration of accelerometer and gyroscope angle data for better angle estimates. The reason for using two sensors even though there is only one relevant physical variable (angle) to be measured is because each type of sensor has its own advantages and disadvantages. By mixing the best parts of each, a better overall estimate of the angle is achieved.

A. Complementary Filter

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Equation (9) is the mathematical implementation of the complementary filter.

\[ \dot{\theta}_{i}(t) = \frac{G_{i}}{G_{\text{gyro}}} \] (7)

\[ \dot{\theta}(t) = \frac{G_{i}}{G_{\text{accel}}} \]

\[ \dot{\theta}_{i}(t) = \frac{G_{i}}{G_{\text{accel}}} \]

Therefore,

\[ \theta_{i}(k) \approx \sum_{k=0}^{n} \dot{\theta}(k)T_{k} \]

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Reference [2] showed the problem with tracking angle measurement and it does not drift while the gyroscope is stationary.

The gyroscope data is integrated at every step, added to the previous angle and then high-pass filtered. After this, it is combined with the low-pass data from the accelerometer. The filter constant, \( \alpha \), is a factor that determines the cutoff time for trusting the gyroscope and filtering in the accelerometer [11]. It can be obtained by first picking an appropriate time constant \( \tau \) for our filter. The relationship between \( \alpha \) and \( \tau \) is shown in (10).

\[ \alpha = \frac{\tau}{\tau + T_{s}} \] (10)

For a time constant of 0.5s, and sampling time of 10ms, \( \alpha = 0.98 \). This is the value of \( \alpha \) used in this work. This implies that for time periods shorter than 0.5s, the gyroscope data takes precedence and the accelerometer data is filtered out while for time periods longer than 0.5s, the accelerometer data takes precedence.

B. Kalman Filter

The Kalman filter was created by Rudolf E. Kalman in 1960. In applying Kalman filtering, one class of measurements defines the basic process equations while other measurements define the measurement equation for the filter [12].

\[ x_{i+1} = A_{i}x_{i} + Bu_{i} + w_{i} \] (11)

\[ y_{i} = Cx_{i} + z_{i} \] (12)

where \( A, B, C \) and \( x \) are \( n \times n \), \( n \times 1 \), \( 1 \times n \) and \( n \times 1 \) matrices, respectively. \( u \) and \( y \) are scalars. \( w_{i} \) and \( z_{i} \) are zero-mean, white, Gaussian noise with covariance \( S_{w} \) and \( S_{z} \) respectively. \( u \) and \( y \) are the noisy measurements representing the gyroscope \( (\dot{\theta}) \) and the accelerometer \( (\theta) \) readings respectively. If \( \theta \) (accelerometer measurement) and \( \dot{\theta}_{i} \) (the amount by which the gyroscope measurement has drifted) are the states of the system, then \( x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \) and \( u = \dot{\theta} \). Consequently,
The process noise covariance matrix \( S_x \) is then the covariance matrix of the state estimate of the accelerometer measurement and the bias. In this case we consider the estimate of the bias and the accelerometer to be independent, so that \( S_x \) is equal to the variance of the bias and the estimate of the accelerometer measurement. That is, \( S_x = \begin{bmatrix} S_y & 0 \\ 0 & S_a \end{bmatrix} \).

Since \( y = \theta \) and \( x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \), then from (12),

\[
A = \begin{bmatrix} 1 & -T_s \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} T_s \\ 0 \end{bmatrix}.
\]

\( \text{diag}(\dot{\theta}_{\text{a}}) \) is the state estimate of the accelerometer measurement and \( \dot{\theta}_{\text{a}} \) is the state estimate of the bias. The values of \( S_y \) and \( S_a \) used in this work are 0.001, 0.003 and 0.03, respectively [13].

V. KALMAN FILTER EQUATIONS

The Kalman filter estimates a process by using a form of feedback control. The filter estimates the state of the process at some time and then obtains feedback in the form of (noisy) measurements. As such, the equations for the Kalman filter fall into two groupings: time update equations and measurement update equations [14]. The current state and error covariance estimates are projected forward by the time update equations to obtain the \( a \text{ priori} \) estimates for the next step (in time). The measurement update equations are responsible for the feedback (comparing the \( a \text{ priori} \) estimate with a new measurement) to obtain an improved \( a \text{ posteriori} \) estimate.

This is shown in Fig. 6.

\[ \hat{x}_{k|k-1} = A\hat{x}_{k-1|k-1} + Bu_{k-1} \]  

That is,

\[
P_{a\text{ priori}} = \begin{bmatrix} 1 & -T_s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} -T_s \\ 0 \end{bmatrix} \hat{\theta}_{\text{a}} \tag{15}
\]

The next thing is to use the previous error covariance matrix \( P_{a\text{ priori}} \) to estimate the \( a \text{ priori} \) error covariance matrix \( P_{a\text{ priori}} \),

\[
P_{k|k-1} = AP_{k-1|k-1}A^T + S_w \tag{16}
\]

That is,
\[
\begin{bmatrix}
P_{00} & P_{01} \\
P_{10} & P_{11}
\end{bmatrix} = \begin{bmatrix}
1 & -T_s & P_{00} & P_{01} \\
0 & 1 & P_{10} & P_{11}
\end{bmatrix} \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} + \begin{bmatrix}
S_y & 0 \\
0 & S_z
\end{bmatrix} T
\]

\[
P \begin{bmatrix}
P_{00} & P_{01} \\
P_{10} & P_{11}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & (1 - K_c) & (1 - K_s) \\
0 & 1 & -K_c & -K_s
\end{bmatrix} \begin{bmatrix}
P_{00} & P_{01} \\
P_{10} & P_{11}
\end{bmatrix}
\]

(27)

\[\text{VI. HUMAN KNEE JOINT ANGLE MEASUREMENT}\]

Human knee joint angle, \(\theta_{\text{knee}}\), is the excursion of the knee joint as shown in Fig. 7 (that is, the movement of the shank relative to the thigh). From Fig. 8, the knee joint angle, \(\theta_k\), can be calculated as;

\[
\theta_k = \theta_t + \theta_s
\]

(28)

where \(\theta_t\) and \(\theta_s\) are the thigh and shank angles, respectively and are both obtained by the data fusion (of accelerometers and gyroscopes data) techniques described earlier.

\[
\begin{bmatrix}
\theta \\
\dot{\theta}
\end{bmatrix} = \begin{bmatrix}
\theta \\
\dot{\theta}
\end{bmatrix} + \begin{bmatrix}
K \hat{h} \\
K \dot{\hat{h}}
\end{bmatrix}
\]

(25)

Lastly, the a posteriori error covariance matrix \(P_{\theta\theta}\) will be updated.

\[
P_{\theta\theta} = (1 - K_c C) P_{\theta\theta\theta}
\]

(26)

The knee joint angle is taken as the knee joint’s excursion starting from an anatomical zero position. The anatomical zero position is the full extension or starting position of the knee joint [15]. One IMU (containing both...
accelerometer and gyroscope) is placed on the thigh to track the thigh movement \( (\theta_I) \) and another is placed on the shank to track shank movement \( (\theta_S) \).

Fig. 9 shows the flowchart used for obtaining and comparing human knee joint angles obtained from Kalman and complementary filters using inertial sensors.

![Flowchart](image)

**VII. REPEATABILITY**

Repeatability defines the usefulness of a measurement system. It is measured by calculating standard deviation [16]. The standard deviation measures the concentration of the data around the mean. The more concentrated, the lower the standard deviation and the higher the repeatability. To consider a measurement system’s repeatability, it must be tested using the same procedure by the same observer, at the same location and within a short period of time. This was done and the result is reported later in the next section.

**VIII. RESULTS**

Data obtained from the continuous monitoring of human knee joint angle using Kalman and complementary filters on accelerometer and gyroscope data during stand-to-squat and walking postures are presented. During the stand-to-squat posture, a subject starts by standing upright, fully extending his knee joint and then squats to full flexion of the knee joint. The subject walks during the walking posture.

A. Results from Complementary Filter

Fig. 10 and Fig. 11 show graphically the results obtained from the complementary filter (with \( \alpha = 0.98 \)) during the aforementioned postures. For the graph in Fig. 10, the subject stood upright, fully extending the knee joint and then squatted (to full flexion). The knee joint angle, which is the total excursion, is 148.20°.

In the graph of Fig. 11, the subject walked. This can be used to check for abnormalities in human gait. If both knee joints of a subject are monitored during a movement, there should be symmetry for normal gait.

Standard and average deviations of knee joint angles obtained over ten trials from the same subject, under the same environmental conditions and the same posture were used for repeatability test. Results are presented in Table I. The mean, standard deviation and average deviation are 149.17°, 0.86° and 0.64°, respectively. The low deviations show high repeatability.

![Graph](image)
Results from Kalman Filter

Fig. 12 and Fig. 13 show graphically the results obtained from the Kalman filter (with the values of $S_a$, $S_b$ and $S_c$ set to 0.001, 0.003 and 0.03, respectively) during the stand-to-squat and walking postures.

Table II shows data obtained over Ten Trials for Stand-to-Squat Posture Using Kalman Filter.

**C. Comparison**

Table III shows data obtained from two subjects during the stand-to-squat posture using complementary filter, Kalman filter and a conventional goniometer. The table shows a very close angle measurement for both complementary and Kalman filters. This means that the complications of Kalman filter can be avoided in this application by implementing the much simple complementary filter.

**TABLE I. DATA OBTAINED OVER TEN TRIALS FOR STAND-TO-SQUAT POSTURE USING COMPLEMENTARY FILTER**

<table>
<thead>
<tr>
<th>Trial</th>
<th>Angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>148.43</td>
</tr>
<tr>
<td>2</td>
<td>148.39</td>
</tr>
<tr>
<td>3</td>
<td>149.37</td>
</tr>
<tr>
<td>4</td>
<td>147.48</td>
</tr>
<tr>
<td>5</td>
<td>149.39</td>
</tr>
<tr>
<td>6</td>
<td>149.29</td>
</tr>
<tr>
<td>7</td>
<td>149.76</td>
</tr>
<tr>
<td>8</td>
<td>149.49</td>
</tr>
<tr>
<td>9</td>
<td>150.60</td>
</tr>
<tr>
<td>10</td>
<td>149.51</td>
</tr>
</tbody>
</table>

**TABLE II. DATA OBTAINED OVER TEN TRIALS FOR STAND-TO-SQUAT POSTURE USING KALMAN FILTER**

<table>
<thead>
<tr>
<th>Trial</th>
<th>Angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>148.69</td>
</tr>
<tr>
<td>2</td>
<td>148.25</td>
</tr>
<tr>
<td>3</td>
<td>149.56</td>
</tr>
<tr>
<td>4</td>
<td>148.96</td>
</tr>
<tr>
<td>5</td>
<td>148.92</td>
</tr>
<tr>
<td>6</td>
<td>148.98</td>
</tr>
<tr>
<td>7</td>
<td>149.80</td>
</tr>
<tr>
<td>8</td>
<td>149.09</td>
</tr>
<tr>
<td>9</td>
<td>148.76</td>
</tr>
<tr>
<td>10</td>
<td>149.29</td>
</tr>
</tbody>
</table>

**TABLE III. DATA OBTAINED FROM COMPLEMENTARY FILTER, KALMAN FILTER AND A CONVENTIONAL GONIOMETER FOR STAND-TO-SQUAT POSTURE**

<table>
<thead>
<tr>
<th></th>
<th>Complementary Filter</th>
<th>Kalman Filter</th>
<th>Goniometer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject 1</td>
<td>161.30</td>
<td>161.75</td>
<td>157.50</td>
</tr>
<tr>
<td>Subject 2</td>
<td>156.88</td>
<td>157.24</td>
<td>155.50</td>
</tr>
</tbody>
</table>

**IX. CONCLUSION**

Each type of data fusion technique described in this work is useable for human knee joint angle tracking because of their high repeatability measures. The Kalman filter is the most appropriate (theoretically) for data fusion of noise measurements but it is computationally intensive. It requires high capacity (processing power) microcontrollers and therefore may not be suitable for low cost applications. The complementary filter on the other hand is simple to implement on any microcontroller thereby aiding low cost designs.

Most of the works presented under review section integrated both accelerometer and gyroscope data using Kalman filter for better angle estimates. The results obtained in this work however shows that the complementary filter works equally well for this application.

**REFERENCES**


Olubiyi O. Akintade obtained his B.Sc. and M.Sc. degrees in Electronic and Electrical Engineering from Obafemi Awolowo University, Ile-Ife, Nigeria, in 2005 and 2015 respectively. He is currently on his Ph.D. programme and lectures in the same Department with interest in Embedded Systems Design for Internet of Things (IoT) Applications. He belongs to the IoT research group of his department. His interests also include Medical Instrumentation and Low Energy Sensor Networks.

Professor Lawrence Kunle Kehinde, a former Engineering Dean and University Deputy Vice Chancellor, received his B.Sc 1st class Hons in Electronics (1971) at Obafemi Awolowo University (OAU), Ile-Ife, Nigeria, and a D.Phil, Control Engineering (1975), at the University of Sussex UK. He had his Post Doctoral Studies in Nuclear Instrumentation at University of California, Berkeley USA (1977-1978) as an IAEA Fellow. He has spent most of his years as a Professor of Instrumentation Engineering at the Obafemi Awolowo University, Ile-Ife, Nigeria. He was the Rector of the first private Polytechnic in Nigeria. A few years ago, he concluded a 3-year Visiting Professor term at the Texas Southern University, Houston Texas USA. He has worked in Techno-Managerial position as the Director of ICT at OAU for years. His major field is Instrumentation Designs and has designed equipment, two of which had received British patents in the past. He has published almost 90 academic publications. He was the founding Principal Investigator of the University’s iLab research and he currently designs remote and virtual experiments for remote experimentation. He is a Chartered Engineer, a Fellow of the Computer Professional Nigeria and a member of IEEE and ASEE.