Gauss Seidal Method to Detect the Lesion with Accuracy Time in Front of Student Test for MR Images

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***Abstract*—In this paper iterative method is proposed with Gauss Seidal (G.S) to detect the position of disease on MR image with less computation time than the statistical approach (student's t test). To solve linear systems, Gauss Seidal build a sequence of approximations that converges to the true solution on other side Student method adjusts a least square model by statistical estimating. Our results obtained with G.S are excellent with accuracy time in front of Student method. The images are obtained from radiology of KOUBA Hospital of Algeria. The software used here is Matlab.**

***Index Terms*—statistical, iterative, Gauss Seidal, student(t)**

I. Introduction

Gauss Seidel is an iterative method, it is known from Liebman method. [1], it is used to solve a linear system of equations. It is named after the German mathematicians Carl Friedrich Gauss and Philipp Ludwig von Seidel [1], and is similar to the Jacobi method. Though it can be applied to any matrix with non-zero elements on the diagonals, convergence is only guaranteed if the matrix is either diagonally dominant, or symmetric and positive definite [1].

A statistically significant t-test result is one in which a difference between two groups is unlikely to have occurred because the sample happened to be atypical. Statistical significance is determined by the size of the difference between the group averages, the sample size, and the standard deviations of the groups [2].

From SPM logiciel which used statistical methods [3].

In this paper we used student technique to detect the position of disease medical images obtained from hospital of Algiers-Algeria with format of "DICOM".

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We have problem of cost computation & accuracy of that we thought for another method which was iterative approach (Gauss Seidal (G.S)) for linear resolution. The results obtained excellent if compared with Student technique.

A. Estimation Statistical

To reach to student t, we need to pass by: matrix of linear model [3]-[8]:

$$Y = X * \beta + \varepsilon \tag{1}$$

$$\begin{pmatrix} y_{11} & .. & .. & y_{1m} \\ . & . & . & . \\ . & . & . & . \\ y_{n1} & . & . & y_{nm} \end{pmatrix} = \begin{pmatrix} x_{11} & .. & .. & x_{1m} \\ . & . & . & . \\ . & . & . & . \\ x_{n1} & . & . & x_{np} \end{pmatrix} * \begin{pmatrix} \beta_{11} & .. & .. & \beta_{1m} \\ . & . & . & . \\ . & . & . & . \\ \beta_{n1} & . & . & \beta_{pm} \end{pmatrix} + \begin{pmatrix} \varepsilon_{11} & .. & .. & \varepsilon_{1m} \\ . & . & . & . \\ . & . & . & . \\ \varepsilon_{n1} & . & . & \varepsilon_{nm} \end{pmatrix} \tag{2}$$

where:
Y: data matrix
X: matrix of conception of n $_x$ p
β: parameter wanted valued of $p \times m$.
ε: the error of n $\times$ m.

The error ϵ is the difference enters the real data and the valued data are:

$$\varepsilon = Y - \tilde{Y} \tag{3}$$

For verify the criteria of the least squares:

$$\xi = \varepsilon * \varepsilon^{T} = \sum_{j=1}^{N} \left[(\varepsilon_{j})^{2} = (Y - \tilde{Y})^{2} \right] \tag{4}$$

T: indicate the working of the transposition

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Derive ϵ by β̃ parameter to get:

$$\frac{\partial \varepsilon}{\partial \tilde{\beta}_m} = 2\sum_{i=1}^{N}\left[(-x_{mj})\left[Y_i - \sum\nolimits_{j=1}^{M}(x_{ji}(\tilde{\beta}_j))\right]\right] = 0$$

or

$$\sum_{i=1}^{N}\left[(-x_{mj})Y_i = \sum_{i=1}^{M} x_{mi}\left[\sum\nolimits_{j=1}^{M}(x_{ji}(\tilde{\beta}_j))\right]\right] = 0 \qquad (5)$$

Least squares estimation is:

$$\tilde{\beta} = X^{-1} * Y = (X^T * X)^{-1} * X^T * Y \qquad (6)$$

After we have the parameter β̃, we pass to calculate test or f-test as follow:

$$t = \frac{\tilde{\beta}}{\sqrt{\dfrac{s * inv(X^t * X)}{n-m}}} \qquad (7)$$

B. Gauss Seidal Estimation

The description of Gauss Seidal method used for given a square system of *n* linear equations as the following [9]-[16]:

$$A * X = b \qquad (8)$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ .. & .. & .. & .. \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} ; X = \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix} ; b = \begin{bmatrix} b_1 \\ b_2 \\ . \\ b_n \end{bmatrix} \qquad (9)$$

The matrix *A* can be decomposed into a diagonal component *D*, and the triangular inferior (*L) & superior* (*U*):

$$A = L + D + U \qquad (10)$$

where

$$D = \begin{bmatrix} a_{11} & 0 & \dots & 0 \\ . & a_{22} & .. & .. \\ 0 & 0 & a_{33} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \dots & a_{nn} \end{bmatrix}; \quad L = \begin{bmatrix} 0 & 0 & \dots & 0 \\ a_{21} & 0 & \dots & 0 \\ a_{31} & a_{32} & 0 & 0 \\ \vdots & . & .. & .. \\ a_{n1} & a_{n2} & \dots a_{n-1m} & 0 \end{bmatrix}, U = \begin{bmatrix} 0 & a_{12} & \dots & a_{1n} \\ 0 & a_{22} & \dots & a_{2n} \\ 0 & 0 & a_{33} & a_{3n} \\ \vdots & . & .. & .. \\ 0 & 0 & \dots 0 & a_{nn} \end{bmatrix} \qquad (11)$$

The result are obtained iteratively via

$$X^{(k+1)} = (D+L)^{-1}.(b - U.X^{(k)}) \qquad (12)$$

The element formula is thus:

$$x_i^{(k+1)} = \frac{1}{a_{ii}}(b_i - \sum\nolimits_{j<i} a_{ij} x_j^{(k+1)} - \sum_{j>i} a_{ij} x_{ij}^{(k)}) \quad ; i, j = 1, 2, \dots n \qquad (13)$$

Note: each element in $\mathbf{x}^{(k)}$ except itself are computed required of $x_i^{(k+1)}$. Unlike the Gauss–Seidel method, we can't overwrite $x_i^{(k)}$ with $x_i^{(k+1)}$, this value will be needed by the rest of the computation. Two vectors of size *n are* the minimum amount of storage.

1) Convergence

The Gauss–Seidel method convergence depends on the matrix *A*. Namely, the procedure is known to converge if either [2]:

- The matrix A is symmetric positive-definite
- The matrix A is strictly or irreducibly diagonally dominant.

Sometimes The Gauss–Seidel method converges even if these conditions are not satisfied.

II. Experimental and Result

The lesion. Student technique have problem of time & diagnostic (see (6)) generally is good for presentation of lesion for small surfaces but without precise, for that we have thought for another method to correct the problem of time, where we proposed the Gauss Seidal method (see (13)).

A. Algorithm

- Read pathological & normal images
- Select the image
- Applied Student equations & Gauss Seidal ones
- Choose precision "e" than compute the error
- If the result is less than the precision then put the result on the pathological image for extract the place of lesion
- Comparison between Student & Gauss Seidal methods.

B. Data Analysis

Our protocol is for patients aged 55 years & 50 years. They made MRI scan with injection of contrast medium. The machine used: type "SIEMENS", and with the field B = 1.5 Tesla. The sequences used here are: T1 and T2.

The results MRI scan of a tumor appear for first patient in the middle of the field on the left side and the third ventricle is to evoke the tumor (see Fig. 1 left). For the second patient the disease appear in the right field. See Fig. 1 (right)

Fig. 1 presents a sample of an image normal in the left & pathological one in the right one for the both models.

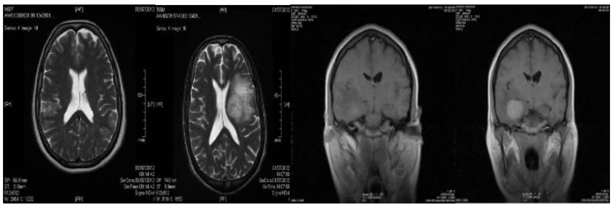

Figure 1. Two models. (left for the both models are normal images; right for the both models are pathological images)

C. Conception and Results

1) Example 1 (EX1)

In the following example we have chosen the precision epsilon 'e' equal to 0.001(e=10^-3).

First patient results: The first patient result is presented in Fig. 2, where Fig. 2b detects with white color the positions of illness (G.S) in 0.127131s (see Fig. 3). On the other hand student presents the place of disease Fig. 2d after 38.28s see (Fig. 4).

Fig. 2c presents the table of t-student [12], where the values of df (degree of freedom) and the probability α (level of signification) are: α=0.005, and df=m=n-1=319. The value of student table equals to 2.82.

Our results (Student test) are compatible with Condition of comparison in hypothesis test to accept or reject values under or above the line (-tab) respectivly which explain the limit of the line (-tab) with red color.

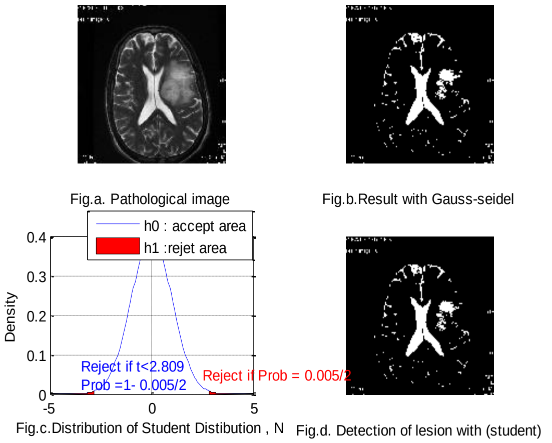

Figure 2. Detection of tumor for first patient where first (a) present a pathological image, in (b) presentation of Gauss Seidal detection, (c) display the distribution of student & (d) present the detection with student method)

```
Elapsed time is 0.127131 seconds.
```

Figure 3. Gauss Seidal (elapsed time)

```
Elapsed time is 38.283398 seconds.
execution of time

N =

   450
```

Figure 4. Elapsed time of student

2) Example 2 (EX2)

Second patient results: For second patient Fig. 5b detect with color white all problems with elapsed time equal to 0.13s (Fig. 6), but Student in Fig. 5d have after 38.71s the (Fig. 7), we haven't got any result which explain the power of our proposition of using iterative methods for medical test. Fig. 5c present the test follows a Student's *t* distribution.

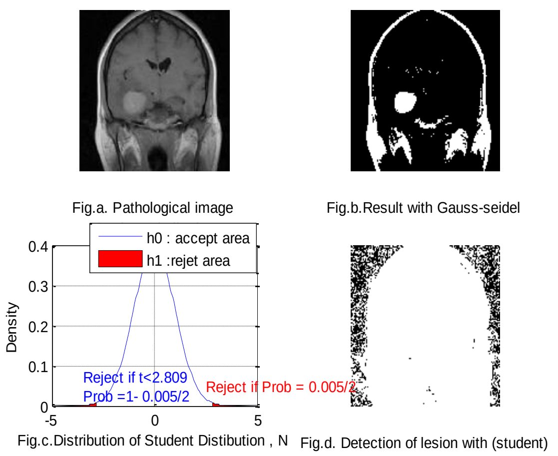

Figure 5. The position of tumor of second patient

```
  450

epsilon =

   1.0000e-003

max =

  200

Elapsed time is 0.131102 seconds.
>>
```

Figure 6. Elapsed time of Gauss Seidal

```
h1 accept
Elapsed time is 38.715993 seconds.
execution of time

N =

   450

df =

   448
```

Figure 7. Elapsed time of student method

3) Example 3 (EX3)

Third patient result: The result for the third patient present always the power of our proposition with G.S method for detection the lesion, where the elapsed time here equal to (0.13s). On the other hand Student didn't execute any lesion after time equal 38s. See Fig. 6, Fig. 7, and Fig. 8). The elapsed time of Gauss Seidal method happened in 0.1325s (Fig. 9), where student execution happened in 38s (Fig. 10).

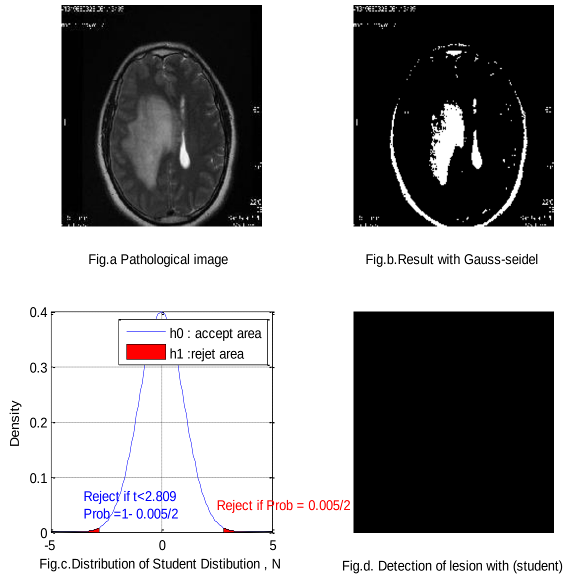

Figure 8. Detection of lesion for third patient

```
n =

  450

epsilon =

   1.0000e-003

max =

  200

Warning: Matrix is close to singular or badly scaled.
         Results may be inaccurate. RCOND = 2.927762e-021.
Warning: Matrix is close to singular or badly scaled.
         Results may be inaccurate. RCOND = 2.927762e-021.
Elapsed time is 0.132546 seconds.
>>
```

Figure 9. Gauss–Seidel execution time

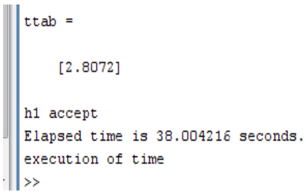

Figure 10. Student time of execution

The following Table I present the results of all precedent examples and display the less time & best execution.

TABLE I. PRESENTATION OUR RESULTS FOR THREE EXAMPLES (IMAGES) WITH ERROR (*E=10^-3*)

Images For e=10^-3	**Time of execution (s) Gauss-Seidel**	**Time of execution (s) Student**	**kind of detection Gauss-Seidel**	**Kind of Detection Student**
Ex1	0.127	38.28	Good	Good
EX2	0.1311	38.71	Good	No result
Ex3	0.13	38.00	well	No result

III. CONCLUSION

In this paper, we have applied two linear resolution methods to extract the place of diseases for MR images. Our results indicate that the iterative Gauss Seidal converges to the solution in less time in comparison with the Statistical method of Student for linear model;

The results of Student technique are plotted in Gaussians curve for α=0.005 and p=1-α, where we concluded:

- If t-calculated or f-calculated are superior to t-table or f-table respectively for a level of significance α=0,005, the hypothesis H0 is rejected.

- If t-calculated or f-calculated are inferior to t-table or f-table respectively for same level of significance (α), the hypothesis H0 is accepted.

As a perspective, we propose to use the same test between Student and Successive over relaxation (SOR).

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