Economic Power Dispatch Problem via Complementarity Problem Approach

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***Abstract*—In this paper, a Complementarity Problem approach to solve the Economic Dispatch Problem (EDP) is presented. The problem is formulated as, Linear Complementarity Problem (LCP) when the transmission losses are neglected and Nonlinear Complementarity Problem (NCP) if the transmission losses are included in the power balance equation. The presented approach is more compact and needs less CPU time compared with other methods and algorithms presented in the literature. The main ideas of this work is presented via numerical examples.**

***Index Terms*—economic dispatch problem, Linear Complementarity Problem (LCP), Nonlinear Complementarity Problem (NCP)**

I. Introduction

Economic Dispatch Problem (EDP) is an important issue in power system operation. The goal of the EDP is to minimize an objective function that reduces the power generation cost while satisfying various physical and operational constraints. The outcome of this approach is expected to improve output scheduling of online units. Under any load condition, EDP determines the power output of each plant (and each generation unit within the plant) which minimizes the overall generation cost [1]. In the traditional EDP, the fuel cost function of a generator has been approximately represented by a quadratic function. Many efforts have been made to solve this problem [2]. Several optimization techniques, such as gradient method, lambda iteration and Newton's methods have been used. In these methods the solution is affected by the incremental cost curves, which are piecewise linear and monotonically increasing, to find the global minimum. Furthermore, recent research [3], [4] proved that although conventional linear programming methods are simple and have high search speed, they have certain drawbacks and limitations.

During the last two decades, the EDP has been solved using several alternative methods such as Artificial Neural Networks [5], [6], Evolutionary Programming (EP) [7]-[9], genetic algorithms (GA) [10], [11], Fast Genetic Algorithm (FGA) [12] Particle Swarm Optimization (PSO) [13], [14].

In this paper, a Complementarity Problem approach to the solution of Economic Dispatch Problem (EDP) with quadratic cost function is presented. The problem formulation is based on the minimization of the generation cost subject to demand and transmission losses constraints and constraints introduced by upper and lower bounds on generated power of each unit. Two cases are considered: i) without transmission losses and ii) subject to transmission losses. In case (i) the problem formulation results finally in a Linear Complementarity Problem (LCP). In case (ii) the problem gets hard nonlinear and its formulated in a compact form as a Nonlinear Complementarity Problem (NCP). For the solution of these types of problems many algorithms and software have been developed recently [15], [16].

The rest of the paper is organized as follows: In section two the problem formulation is shown, section three outlines numerical examples that illustrate the performance of the suggested approach in comparison with other methods, finally the conclusion is discussed in section four.

II. Problem Formulation

The economic dispatch problem is a mathematical optimization problem. The goal of the solution is to achieve the optimal power dispatches from various power generating units for a determined time period to minimize the total generation cost while satisfying the specified constraint. The most commonly minimized function is the total cost of real power generation. The individual costs are assumed to be functions of active power generation and represented by quadratic equations. The constraints

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are the power balance equation and the upper and lower limits on the generated power by each unit.

A. Economic Dispatch without Transmission Losses

In this case the problem is to minimize the objective function subject to constraints of the demand load and minimum and maximum limits on the power output of the units. The mathematical optimization problem is as follows:

$$\min F_t = \sum_{i=1}^{n} F_i(P_i) = \sum_{i=1}^{n} a_i P_i^2 + b_i P_i + c_i \quad (1)$$

Subject to:

$$\sum_{i=1}^{n} P_i = P_D \quad (2)$$

$$P_{i,min} \le P_i \le P_{i,max} \text{ for } i = 1,2,\ldots,n \quad (3)$$

where:
i unit number
n number of units
$F_i(.)$ fuel cost function of the unit i
P_i power from the i^{th} generator
P_D demand load
$P_{i,max}$ upper limit of the i^{th} generator
$P_{i,min}$ lower limit of the i^{th} generator

Equation (3) can be written in the following form [17]:

$$P_i^{'} = P_i - P_{i,min} \ge 0, \quad P_i^{''} = P_{i,max} - P_i \ge 0 \quad (4)$$

Adding these equations we obtain:

$$P_i^{'} + P_i^{''} = P_{i,max} - P_{i,min} \quad (5)$$

and since $P_i^{''} \ge 0$ this equation can be written in term of $P_i^{'}$:

$$P_i^{'} \le P_{i,max} - P_{i,min} \quad (6)$$

The first equation in (4) can be written in terms of the new variable $P_i^{'}$ to obtain the power from the i^{th} generator:

$$P_i = P_i^{'} + P_{i,min} \quad (7)$$

Substituting (7) in (1) and (2) the optimization problem can be expressed in terms of $P_i^{'}$ as follows:

$$\text{minimize } F_t = \sum_{i=1}^{n} F_i(P_i^{'} + P_{i,min}) \quad (8)$$

Subject to the following constraints:

$$\sum_{i=1}^{n} P_i^{'} = P_D - \sum_{i=1}^{n} P_{i,min} \quad (9)$$

$$P_i^{'} \ge 0 \quad (10)$$

Constraint (9) can be written in the form of two inequality constraints:

$$\sum_{i=1}^{n} P_i^{'} \le P_D - \sum_{i=1}^{n} P_{i,min} \quad (11)$$

$$-\sum_{i=1}^{n} P_i^{'} \le -P_D + \sum_{i=1}^{n} P_{i,min}$$

The problem described by the (8), (10), and (11) is an optimization problem, which contains both equality and inequality constrains, needs the deployment of the Karush-Kuhn-Tucker (KKT) optimality conditions [18]. Therefore these equations are rewritten in matrix form as follows:

$$\min F_t = (\mathbf{P}^{'})^t \mathbf{D}\mathbf{P}^{'} + \mathbf{g}^t \mathbf{P}^{'} + \mathbf{k}^t \quad (12)$$

subject to:

$$\mathbf{e}\mathbf{P}^{'} \le P_D - \mathbf{e}\mathbf{P}_{min} \quad (13)$$

$$-\mathbf{e}\mathbf{P}^{'} \le -P_D + \mathbf{e}\mathbf{P}_{min}$$

$$\mathbf{I}\mathbf{P}^{'} \le \mathbf{P}_{max} - \mathbf{P}_{min}$$

$$\mathbf{P}^{'} \ge \mathbf{0}$$

here

$$\mathbf{D} = \begin{bmatrix} a_1 & 0 & \ldots & 0 \\ 0 & a_2 & \ldots & 0 \\ \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & \ldots & a_n \end{bmatrix}, \ \mathbf{g} = \begin{bmatrix} b_1 + 2a_1 P_{1,min} \\ b_2 + 2a_2 P_{2,min} \\ \vdots \\ b_n + 2a_n P_{n,min} \end{bmatrix}$$

$$\text{and } \mathbf{k} = \begin{bmatrix} c_1 + b_1 P_{1,min} + a_1 P_{1,min}^2 \\ c_2 + b_2 P_{2,min} + a_2 P_{2,min}^2 \\ \vdots \\ c_n + b_n P_{n,min} + a_n P_{n,min}^2 \end{bmatrix}$$

$\mathbf{e}$ is $1 \times n$ is vector of ones, $\mathbf{I}$ is $n \times$ n identity matrix, $\mathbf{P}^{'} = [P_1^{'}, P_2^{'}, \ldots, P_n^{'}]^t$, $\mathbf{P}_{min} = [P_{1,min}, P_{2,min}, \ldots, P_{n,min}]^t$ and $\mathbf{P}_{max} = [P_{1,max}, P_{2,max}, \ldots, P_{n,max}]^t$.

The Karush-Kuhn-Tucker (KKT) Lagrange for the dispatch problem given by (12) and (13) is [19]:

$$L = (\mathbf{P}^{'})^t \mathbf{D}\mathbf{P}^{'} + \mathbf{g}^t \mathbf{P}^{'} + \mathbf{k}^t + \boldsymbol{\lambda}^t (\mathbf{e}\mathbf{P}^{'} - P_D + \mathbf{e}\mathbf{P}_{min}) + \boldsymbol{\mu}^t (-\mathbf{e}\mathbf{P}^{'} + P_D - \mathbf{e}\mathbf{P}_{min}) + \mathbf{u}^t (\mathbf{I}\mathbf{P}^{'} - \mathbf{P}_{max} + \mathbf{P}_{min}) + \mathbf{y}^t (-\mathbf{P}^{'}) \quad (14)$$

where $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$, $\mathbf{u}$, and $\mathbf{y}$ are the Lagrange multipliers associated with the constraints. The Karush-Kuhn-Tucker (KKT) necessary optimality conditions for the above problem are:

$$\frac{\partial L}{\partial \mathbf{P}^{'}} = 2\mathbf{D}\mathbf{P}^{'} + \mathbf{g} + \mathbf{e}^t \boldsymbol{\lambda} - \mathbf{e}^t \boldsymbol{\mu} + \mathbf{I}\mathbf{u} - \mathbf{y} = \mathbf{0} \quad (15)$$

$$\frac{\partial L}{\partial \boldsymbol{\lambda}} = \mathbf{eP}' - P_D + \mathbf{eP}_{min} \leq 0$$

$$\frac{\partial L}{\partial \boldsymbol{\mu}} = -\mathbf{eP}' + P_D - \mathbf{eP}_{min} \leq 0$$

$$\frac{\partial L}{\partial \mathbf{u}} = \mathbf{IP}' - \mathbf{P}_{max} + \mathbf{P}_{min} \leq \mathbf{0}$$

Introducing the nonnegative slack variables v^+, v^-, and $\boldsymbol{\eta}$ the previous KKT conditions can be written as:

$$\mathbf{y} - 2\mathbf{DP}' - \mathbf{e}^t\boldsymbol{\lambda} + \mathbf{e}^t\boldsymbol{\mu} - \mathbf{Iu} = \mathbf{g} \quad (16)$$
$$v^+ + \mathbf{eP}' = P_D - \mathbf{eP}_{min}$$
$$v^- - \mathbf{eP}' = -P_D + \mathbf{eP}_{min}$$
$$\boldsymbol{\eta} + \mathbf{IP}' = \mathbf{P}_{max} - \mathbf{P}_{min}$$

Comparing (16) with (5) we conclude that $\boldsymbol{\eta} = \mathbf{P}'' = [P_1'', P_2'', \ldots, P_n'']$. Substituting in the above equation we obtain:

$$\mathbf{y} - 2\mathbf{DP}' - \mathbf{e}^t\boldsymbol{\lambda} + \mathbf{e}^t\boldsymbol{\mu} - \mathbf{Iu} = \mathbf{g} \quad (17)$$
$$v^+ + \mathbf{eP}' = P_D - \mathbf{eP}_{min}$$
$$v^- - \mathbf{eP}' = -P_D + \mathbf{eP}_{min}$$
$$\mathbf{P}'' + \mathbf{IP}' = \mathbf{P}_{max} - \mathbf{P}_{min}$$

where, the nonnegative variables fullfil the following complementarity conditions:

$$\mathbf{y} \geq \mathbf{0}, \quad \mathbf{P}' \geq \mathbf{0}, \quad \mathbf{y}^t\mathbf{P}' = 0 \quad (18)$$
$$v^+ \geq 0, \quad \boldsymbol{\lambda} \geq 0, \quad v^{+t}\boldsymbol{\lambda} = 0$$
$$v^- \geq \mathbf{0}, \quad \boldsymbol{\mu} \geq 0, \quad v^{-t}\boldsymbol{\mu} = 0$$
$$\mathbf{u} \geq \mathbf{0}, \quad \mathbf{P}'' \geq \mathbf{0}, \quad \mathbf{u}^t\mathbf{P}'' = 0$$

The system of (17) and (18) is a standard Linear Complementarity Problem (LCP) which can be written in the following form:

$$\mathbf{w} - \mathbf{Mz} = \mathbf{q} \quad (19)$$
$$\mathbf{w} \geq \mathbf{0}, \quad \mathbf{z} \geq \mathbf{0}, \quad \mathbf{w}^t\mathbf{z} = 0$$

where

$$\mathbf{M} = \begin{bmatrix} 2\mathbf{D} & \mathbf{e}^t & -\mathbf{e}^t & \mathbf{I} \\ -\mathbf{e} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{e} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \ \mathbf{q} = \begin{bmatrix} \mathbf{g} \\ P_D - \mathbf{eP}_{min} \\ -P_D + \mathbf{eP}_{min} \\ \mathbf{P}_{max} - \mathbf{P}_{min} \end{bmatrix},$$

$$\mathbf{w} = \begin{bmatrix} \mathbf{y} & v^+ & v^- & \mathbf{P}'' \end{bmatrix}^t \text{ and } \mathbf{z} = \begin{bmatrix} \mathbf{P}' & \boldsymbol{\lambda} & \boldsymbol{\mu} & \mathbf{u} \end{bmatrix}^t$$

B. Economic Dispatch with Transmission Lines Losses

The EDP is considered here with the presence of transmission losses. Equation (2) is modified to account for such losses as follows:

$$\sum_{i=1}^{n} P_i = P_D + P_L \quad (20)$$

where P_L is the total transmission loss and is given by the following equation [1]:

$$P_L = \sum_{i=1}^{n}\sum_{j=1}^{n} P_i B_{ij} P_j + \sum_{i=1}^{n} B_{0i} P_i + B_{00} \quad (21)$$

Using (7) and (21) in (20) the power balance constraint reduces to:

$$\mathbf{eP}' - \left\{(\mathbf{P}')^t\mathbf{B} + 2\mathbf{P}^t_{min}\mathbf{B} + \mathbf{B}_0\right\}\mathbf{P}' \quad (22)$$
$$= P_D - \mathbf{eP}_{min} + \mathbf{P}^t_{min}\mathbf{BP}_{min}$$
$$+\mathbf{B}_0\mathbf{P}_{min} + \mathbf{B}_{00}$$

where $\mathbf{B}$, $\mathbf{B}_0$, and $\mathbf{B}_{00}$ represent the B-coefficients of transmission loss formula.

Equation (22) can be written in a more compact form as follows:

$$\left\{\mathbf{e} - (\mathbf{P}')^t\mathbf{B} - \mathbf{V}\right\}\mathbf{P}' = \mathbf{Q} \quad (23)$$

here $\mathbf{e}$ is $1 \times n$ is vector of ones, and $\mathbf{V} = 2\mathbf{N} + \mathbf{B}_0$ where $\mathbf{N} = \mathbf{P}^t{}_{min}\mathbf{B}$ and

$$\mathbf{Q} = P_D - \mathbf{eP}_{min} + \left\{\mathbf{N} + \mathbf{B}_0\right\}\mathbf{P}_{min} + \mathbf{B}_{00}$$

Equality constraint given by (23) can be replaced by two inequality constraints

$$\left\{\mathbf{e} - (\mathbf{P}')^t\mathbf{B} - \mathbf{V}\right\}\mathbf{P}' \leq \mathbf{Q} \quad (24)$$
$$-\left\{\mathbf{e} - (\mathbf{P}')^t\mathbf{B} - \mathbf{V}\right\}\mathbf{P}' \leq -\mathbf{Q}$$

The Karush-Kuhn-Tucker (KKT) Lagrangian for the dispatch problem defined by the (5), (10), (12), and (24) is:

$$L = (\mathbf{P}')^t\mathbf{DP}' + \mathbf{g}^t\mathbf{P}' + \mathbf{k}^t \quad (25)$$
$$+\boldsymbol{\lambda}^t\left[\left\{\mathbf{e} - (\mathbf{P}')^t\mathbf{B} - \mathbf{V}\right\}\mathbf{P}' - \mathbf{Q}\right]$$
$$-\boldsymbol{\mu}^t\left[\left\{\mathbf{e} - (\mathbf{P}')^t\mathbf{B} - \mathbf{V}\right\}\mathbf{P}' - \mathbf{Q}\right]$$
$$+\mathbf{u}^t(\mathbf{IP}' - \mathbf{P}_{max} + \mathbf{P}_{min}) + \mathbf{y}^t(-\mathbf{P}')$$

where $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$, $\mathbf{u}$, and $\mathbf{y}$ are the Lagrange multipliers associated with the constraints. The KKT necessary optimality conditions for the above problem are:

$$\frac{\partial L}{\partial \mathbf{P}'} = 2\mathbf{DP}' + \mathbf{g} \quad (26)$$
$$+\left\{\mathbf{e} - 2(\mathbf{P}')^t\mathbf{B} - \mathbf{V}\right\}^t\boldsymbol{\lambda}$$
$$-\left\{\mathbf{e} - 2(\mathbf{P}')^t\mathbf{B} - \mathbf{V}\right\}^t\boldsymbol{\mu} + \mathbf{u} - \mathbf{y} = \mathbf{0}$$
$$\frac{\partial L}{\partial \boldsymbol{\lambda}} = \left\{\mathbf{e} - (\mathbf{P}')^t\mathbf{B} - \mathbf{V}\right\}\mathbf{P}' - \mathbf{Q} \leq 0$$
$$\frac{\partial L}{\partial \boldsymbol{\mu}} = -\left\{\mathbf{e} - (\mathbf{P}')^t\mathbf{B} - \mathbf{V}\right\}\mathbf{P}' + \mathbf{Q} \leq 0$$
$$\frac{\partial L}{\partial \mathbf{u}} = \mathbf{IP}' - \mathbf{P}_{max} + \mathbf{P}_{min} \leq \mathbf{0}$$

Introducing the nonnegative slack variables $\mathbf{v}^+$, $\mathbf{v}^-$, and η these KKT conditions can be written as:

$$\mathbf{y}-2\mathbf{D}\mathbf{P}'-\left\{\mathbf{e}-2(\mathbf{P}')^t\mathbf{B}-\mathbf{V}\right\}^t\lambda \tag{27}$$

$$+\left\{\mathbf{e}-2(\mathbf{P}')^t\mathbf{B}-\mathbf{V}\right\}^t\mu-\mathbf{u}=\mathbf{g}$$

$$\mathbf{v}^+ +\left\{\mathbf{e}-(\mathbf{P}')^t\mathbf{B}-\mathbf{V}\right\}\mathbf{P}'=\mathbf{Q}$$

$$\mathbf{v}^- -\left\{\mathbf{e}-(\mathbf{P}')^t\mathbf{B}-\mathbf{V}\right\}\mathbf{P}'=-\mathbf{Q}$$

$$\eta+\mathbf{I}\mathbf{P}'=\mathbf{P}_{max}-\mathbf{P}_{min}$$

Substituting η with $\mathbf{P}''$ we obtain:

$$\mathbf{y}-2\mathbf{D}\mathbf{P}'-\left\{\mathbf{e}-2(\mathbf{P}')^t\mathbf{B}-\mathbf{V}\right\}^t\lambda \tag{28}$$

$$+\left\{\mathbf{e}-2(\mathbf{P}')^t\mathbf{B}-\mathbf{V}\right\}^t\mu-\mathbf{u}=\mathbf{g}$$

$$\mathbf{v}^+ +\left\{\mathbf{e}-(\mathbf{P}')^t\mathbf{B}-\mathbf{V}\right\}\mathbf{P}'=\mathbf{Q}$$

$$\mathbf{v}^- -\left\{\mathbf{e}-(\mathbf{P}')^t\mathbf{B}-\mathbf{V}\right\}\mathbf{P}'=-\mathbf{Q}$$

$$\mathbf{P}''+\mathbf{I}\mathbf{P}'=\mathbf{P}_{max}-\mathbf{P}_{min}$$

The nonnegative variables fullfil the following complementarity conditions:

$$\mathbf{y}\geq\mathbf{0},\quad \mathbf{P}'\geq\mathbf{0},\quad \mathbf{y}^t\mathbf{P}'=0 \tag{29}$$

$$v^+\geq\mathbf{0},\quad \lambda\geq\mathbf{0},\quad (v^+)^t\lambda=0$$

$$v^-\geq\mathbf{0},\quad \mu\geq\mathbf{0},\quad (v^-)^t\mu=0$$

$$\mathbf{P}''\geq\mathbf{0},\quad \mathbf{u}\geq\mathbf{0},\quad (\mathbf{P}'')^t\mathbf{u}=0$$

The system of (28) and orthogonality conditions (29) form a standard Nonlinear Complementarity Problem (NCP) which can be written as follows:

$$\mathbf{z}\geq\mathbf{0},\quad \mathbf{M}(\mathbf{z})\geq\mathbf{0},\quad \mathbf{z}^t\mathbf{M}(\mathbf{z})=0 \tag{30}$$

where

$$\mathbf{z}=\left[\mathbf{P}'\ \ \lambda\ \ \mu\ \ \mathbf{u}\right]^t$$

III. Numerical Examples

In this section, numerical examples are given to illustrate the developed method. The formulated LCP can be solved using iterative methods, Newton methods, and pivoting methods [20]. Here the LCP problem is solved using the MATLAB code pathlcp.m (written and maintained jointly by Michael C. Ferris and Madison and Todd Munson) [15]. On the other hand the formulated Nonlinear Complementarity Problem NCP can be solved using different algorithms addressed in the literature, in this work the MATLAB LMMCP.M developed and written by Kanzow and Petra is used [16]. Kanzow and Petra reformulated the NCP to nonlinear Least Square method and then they apply a Levenberg-Marquardt-type method to find the solution.

A. Examples without Transmission Lines Losses

1) First test system

The first system has six thermal units with the following cost functions and their associated constraints for each unit:

$$F_1=0.001562P_1^2+7.92P_1+561$$
$$100\leq P_1\leq 600$$

$$F_2=0.00194P_2^2+7.85P_2+310$$
$$100\leq P_2\leq 400$$

$$F_3=0.00482P_3^2+7.97P_3+78$$
$$50\leq P_3\leq 200$$

$$F_4=0.00139P_4^2+7.06P_4+500$$
$$140\leq P_4\leq 590$$

$$F_5=0.00184P_5^2+7.46P_5+295$$
$$110\leq P_5\leq 400$$

$$F_6=0.00184P_6^2+7.46P_3+295$$
$$110\leq P_6\leq 400$$

Table I shows the results of the proposed method compared with a fuzzy logic controlled genetic algorithm (FCGA) [10] and a genetic algorithm based on arithmetic crossover (GAAC) [11] when the load demand was 1800 MW. Its clearly evident from this table that the solution of the LCP results in terms of operation costs is exactly the same as that obtained by the GAAC method, while the FCGA method produces higher operation cost. In general, strong correlation is evident between results obtained from all these methods.

2) Second test system

The second test system has thirteen generation units. The data of each generation unit are shown in Table II. This test system is solved for different values of demand load ranging between 550 MW and 2960 MW. The individual generated power, minimum fuel cost, the total generated power, and the CPU time are shown in Table III.

TABLE I. Results of FCGA, GAAC, and LCP for 6-Unit System

Method	Load	P_1 (MW)	P_2 (MW)	P_3 (MW)	P_4 (MW)	P_5 (MW)	P_6 (MW)	F_t (\$/h)
FCGA	1800	250.49	215.43	109.92	572.84	325.66	325.66	16585.85
GAAC	1800	248.14	217.74	75.20	587.8	325.56	325.56	16579.33
LCP	1800	247.9995	217.7192	75.1816	588.0397	325.53	325.53	16579.33

TABLE II. GENERATION UNIT'S DATA

Unit No.	P_{min} (MW)	P_{max} (MW)	Cost Coefficients		
			a ($/MW²h)	b ($/MWh)	c ($/h)
1	0	680	0.00028	8.1	550
2	0	360	0.00056	8.1	309
3	0	360	0.00056	8.1	307
4	60	180	0.000324	7.74	240
5	60	180	0.000324	7.74	240
6	60	180	0.000324	7.74	240
7	60	180	0.000324	7.74	240
8	60	180	0.000324	7.74	240
9	60	180	0.000324	7.74	240
10	40	120	0.000284	8.6	126
11	40	120	0.000284	8.6	126
12	55	120	0.000284	8.6	126
13	55	120	0.000284	8.6	126

TABLE III. RESULTS USING LCP FOR 13-UNIT TEST SYSTEM

P_d (MW)	550	1350	1800	2200	2960
P_1	0	64.8369	108.4478	306.2611 9	680
P_2	0	60.8917	106.8636	290.6486	360
P_3	0	60.8917	106.8636	290.6486	360
P_4	60	156.7656	178.1567	179.9999	180
P_5	60	156.7656	178.1567	179.9999	180
P_6	60	156.7656	178.1567	179.9999	180
P_7	60	156.7656	178.1567	179.9999	180
P_8	60	156.7656	178.1567	179.9999	180
P_9	60	156.7656	178.1567	179.9999	180
P_{10}	40	156.7656	178.1567	179.9999	180
P_{11}	40	54.9446	102.0583	58.2402	120
P_{12}	55	54.9446	102.0583	58.2402	120
P_{13}	55	54.9446	102.0583	58.2402	120
Total cost ($/h)	7540.0254	13874.4067	17599.2799	20845.1192	27291.1680
Total P (MW)	550	1349.9997	1800	2200	2960
CPU time (s)	0.003744 6	0.010296	0.011076	0.010296	0.007332

B. Examples with Transmission Losses

Here, a standard IEEE 30 bus test system is used as a third test case. The generation unit's data in the IEEE 30 bus test system are given in Table IV. The solution of the obtained NCP produces the amount of the generated power of each unit that results in the minimum generation cost. As mentioned previously the MATLAB program (LMMCP.M) written by Kanzow and Petra [16] is used to solve the problem. The obtained results for different load demand are shown in Table V.

The results of the proposed formulation of the IEEE 30 bus system are compared with other methods such as Evolutionary Programming IDE [7] and EP-OPF [8], fast genetic algorithm approach (FGA) [12], Pattern Search (PS) and a combination of Genetic Algorithm and Patern Search (GA-PS) [21], and Hybrid Self-adaptive Differential Evolution Methods SADE-ALM [22]. The load demand was 283.4 MW and the obtained results are shown in Table VI.

TABLE IV. IEEE 30 BUS TEST SYSTEM GENERATION UNIT'S DATA

Unit No.	P_2	P_2	Cost Coefficients		
			a ($/MW²h)	b ($/MWh)	c ($/h)
1	50	200	0.00375	2	0
2	20	80	0.01750	1.75	0
3	15	50	0.06250	1	0
4	10	35	0.00834	3.25	0
5	10	30	0.02500	3	10
6	12	40	0.02500	3	0

TABLE V. RESULTS USING NCP FOR IEEE 30 BUS TEST SYSTEM

P_d (MW)	117	170	220	283.4	330	380	435
P_1	50.9986	96.6509	137.0558	176.2631	198.2098	200	200
P_2	20	29.3137	38.8954	48.3829	53.7786	71.7666	80
P_3	15	15	17.7638	20.8706	22.7593	28.4625	49.9150
P_4	10	10	10	22.7129	34.9339	35	35
P_5	10	10	10	12.4533	16.6445	30	30
P_6	12	12	12	12	15.4892	28.4646	40
Total cost ($/h)	288.2469	429.1655	582.0141	801.7211	976.6427	1186.9352	1404.1008
Total P (MW)	117.9986	172.9647	225.7149	292.6829	341.8153	393.6937	434.9150
P_L (MW)	0.9986	2.9647	5.7149	9.2829	11.8153	13.6937	14.9150
CPU time (s)	0.015444	0.012792	0.016536	0.019188	0.021996	0.020748	0.021372

TABLE VI. RESULTS OF IDE, EP-OPF, FGA, PS, GA-PS, SADE-ALM AND NCP FOR 6-UNIT SYSTEM

Method	P_1 (MW)	P_2 (MW)	P_3 (MW)	P_4 (MW)	P_5 (MW)	P_6 (MW)	F_t (MW)	P_L ($/h)	CPU Time
IDE	181.6329	50.12272	20.15867	10	10.46971	12	794.9129	9.30433	1.466409
EP-OPF	173.848	49.998	21.386	22.630	12.928	12	802.404	9.4791	NA
FGA	189.613	47.745	19.5761	13.8752	10	12	799.823	9.6897	0.125
PS	175.727	48.6812	21.4282	22.8313	12.0667	12	802.015	9.3349	NA
GA-PS	175.6627	48.6413	21.4222	22.6219	12.3806	12	802.0138	9.3286	NA
SADE-ALM	176.1522	48.8391	21.5144	22.1299	12.2435	12	802.404	9.491	NA
NCP	176.2631	48.3829	20.8706	22.7129	12.4533	12	801.7211	9.2829	0.0192

IV. CONCLUSION

In this paper a new approach for the formulation of the economic power dispatch problem is presented. The problem without transmission losses taken into account in the power balance equation is formulated as Linear Complementarity Problem (LCP). Taking into account transmission losses the problem gets hard nonlinear, and a Nonlinear Complementarity Problem (NCP) is formulated to present an efficient solution. Recently many efficient algorithms and software have been devised and developed. Numerical examples given section 3 shows that, in comparison with other methods, the presented approach provides an efficient, time saving, highly accurate, and cost effective technique with evident strong correlation with other well known established techniques.

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