Recursive Estimation of Emitter Position Using AOA Measurements

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***Abstract*—The concept of geolocation is a real position estimation method of a specific object which generates a radio signal. Angle of Arrival (AOA) scheme is commonly used to discover the position of active radio sources or any military radio emitter. In two dimensional space, the geolocation method based on AOA scheme uses at least two directional data of propagation signal from receivers for the position estimation of an emitter. In AOA based geolocation, the measurement error which is caused by environmental noise is a major factor of estimation accuracy decrease. Therefore, this paper introduces the precise geolocation method using AOA measurement data through the Recursive Weighted Least Square (RWLS) scheme. The RWLS scheme can obtain the iterative solution of emitter position which is more accurate than the general Least Square (LS) solution with weighting measurement data. For the performance verification, some simulation results demonstrate that our proposed RWLS scheme improves the accuracy and computational speed for geolocation.**

***Index Terms*—angle of arrival, estimation accuracy, geolocation, recursive weighted least square**

I. Introduction

It is widely known that many industrial fields such as mobile communication, navigation, and augmented reality platform have a great interest in geolocation using a radio signal. Although the most widely used geolocation method is the Global Positioning System (GPS), there are many disruptive factors such as jamming which causes the decrease of position estimation accuracy. GPS has a limitation for geolocation in indoor environment or downtown since GPS necessarily requires a satellite signal. Recently, the geolocation methods based on Time of Arrival (TOA), time difference of arrival (TDOA), and Received Signal Strength (RSS) measurements have been globally issued since the separately distributed receivers get a signal relatively stronger than GPS [1]-[3]. In this paper, we propose an AOA measurement based geolocation scheme in order to determine the position of an emitter. Among different kinds of the radio signal based geolocation methods, the position estimation using AOA measurement data is a widely used method with active radio signal source. The AOA measurements of the transmitted signal from an emitter can be acquired from multiple antenna arrays in receivers. The line of bearing from emitter to receiver can be drawn with measured AOA data. Therefore, the estimated position of emitter can be computed from the intersection area of at least two lines of bearing in two dimensional space [4]-[6].

In the geolocation using an active radio source, environmental noise is the major factor which causes the estimation inaccuracy of position estimation. In order to improve the estimation accuracy of geolocation, Wang [7] proposed the Weighted Least Square (WLS) scheme based approach to the localization problem in wireless sensor networks using RSS measurements. Jung [8] suggested an indoor localization method based on TDOA using LED ceiling lamps. A unique frequency address was assigned to each LED lamp, and it was used to identify each signal. A mobile emitter geolocation method using Gaussian mixture presentation of measurements-integrated track splitting filter is proposed by Musicki [9].

In the AOA data based geolocation, the measurement error which is caused by environmental noise disrupts to estimate the precise position of emitter. The measurement error causes the uncertainty of drawn lines of bearing from each receiver. The wrong drawn AOA lines by measurement error cannot intersect at an exact point [10]. In order to compensate the measurement error and estimate a precise position of emitter, we suggest the RWLS algorithm based optimization scheme. RWLS scheme is the recursive approach of WLS algorithm which gives weight to the measurement AOA data in accordance of its significance. RWLS scheme is applied to the geolocation formula which can be acquired using the measured angle data from each receiver. The rest of this paper is organized as follows. In Section II, the NLOS signal detection algorithm using Kalman filter is derived. Using identified LOS data, the AOA measurement based geolocation formula is established to derive the objective function which denotes the error of position estimation in Section III. Section IV explains the recursive optimal solution using RWLS in order to minimize the estimation error and decrease the calculation rate. In Section V, some simulation results confirm the effectiveness of our proposed RWLS algorithm approach. Conclusion is drawn in Section VI.

II. NLOS Signal Detection Using Kalman Filter

In position based services, NLOS noise is one of the significant problems which cause an incidence angle

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difference of received signal traveling from an emitter to a receiver. The wireless signal is obstructed and reflected by the obstacles such as walls which locate between an emitter and receivers and becomes NLOS signal. In Section II, the TOA data which is measured at each receiver is divided by some sections according to the sampling time. In these sections, the NLOS noise contained data sections cause the inaccuracy in position determination. In this paper, the NLOS noise contained data section can be identified using a Kalman filter based hypothesis test. In order to apply a Kalman filter to a state of emitter, the state space model of an emitter motion needs to be formulated. The state equations are independently derived for the *x*-direction and *y*-direction of an emitter as follows

$$\mathbf{s}_{k+1} = \mathbf{\Theta}\mathbf{s}_k + \mathbf{\Phi}\rho_k \tag{1}$$

where

$$\mathbf{\Theta} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix},\ \mathbf{\Phi} = \begin{bmatrix} 0 \\ \Delta t \end{bmatrix}$$

The state vector $\mathbf{s}_k$ is denoted as $[l_x\ v_x]^T$ for the *x*-direction or $[l_y\ v_y]^T$ for the *y*-direction at time t_k. l_x and v_x are the position and velocity in the *x*-direction, respectively. ρ_k is the driving variable with a variance $C_\rho = \sigma_\rho^2$. The measurement state variable m_k is expressed as follows

$$m_k = \mathbf{\Pi}\mathbf{s}_k + \mu_k \tag{2}$$

with $\mathbf{\Pi} = [1\quad 0]$. m_k which is used as an observation factor is the measurement noise variable with a variance of $C_\mu = \sigma_\mu^2$.

The iterative process of the Kalman filter can be derived as follows

$$\begin{aligned} \hat{\mathbf{s}}_k^- &= \mathbf{\Theta}\hat{\mathbf{s}}_{k-1} \\ \mathbf{R}_k^- &= \mathbf{\Theta}\mathbf{R}_{k-1}\mathbf{\Theta}^T + \mathbf{\Phi}C_\rho\mathbf{\Phi}^T \\ \mathbf{K}_k &= \mathbf{R}_k^-\mathbf{\Pi}^T(\mathbf{\Pi}\mathbf{R}_k^-\mathbf{\Pi}^T + C_\mu)^{-1} \\ \hat{\mathbf{s}}_k &= \hat{\mathbf{s}}_k^- + \mathbf{K}_k(m_k - \mathbf{\Pi}\hat{\mathbf{s}}_k^-) \\ \mathbf{R}_k &= \mathbf{R}_k^- - \mathbf{K}_k\mathbf{\Pi}\mathbf{R}_k^- \end{aligned} \tag{3}$$

$\mathbf{R}_k$ represents the covariance matrix of a state vector $\mathbf{s}_k$. $\mathbf{K}_k$ denotes a Kalman filter gain. The position estimation of emitter at the time t_k, denoted by $\hat{\mathbf{s}}_k$, can be derived through the (3).

The measured distance data between an emitter and each receiver is compared with estimated distance which can be obtained in (3) in order to identify whether the section is under LOS or NLOS condition. The distance data can be calculated with the measured TOA data and the propagation speed of signal.

The measured distance data $d_m(t_k)$ composes of real distance $d_m^0(t_k)$, a measurement error $\Delta d_m(t_k)$, and an NLOS error $\Delta d_{nlos}(t_k)$ as below

$$d_m(t_k) = d_m^o(t_k) + \Delta d_m(t_k) + \Delta d_{nlos}(t_k) \tag{4}$$

We can obtain the estimated location of an emitter by using the Kalman filter which is represented by equation (3). The standard deviation of measured distance can be formulated as follows

$$\hat{\sigma}_{nlos} = \sqrt{\frac{1}{N}\sum_{k=1}^{N}(d_m(t_k) - d_{Kalman}(t_k))^2} \tag{5}$$

where $d_{kalman}(t_k)$ is the estimated distance which can be calculated with the estimated state of an emitter. N is the number of distance data $d_m(t_k)$ in each partition.

In this paper, one receiver is supposed to be located in the LOS environment for identification of NLOS noise contained section. The section that contains the NLOS noise can be identified using the following hypothesis.

$$\begin{aligned} H_1(LOS\ condition):&\ \hat{\sigma}_{nlos} < \varepsilon\sigma_m \\ H_2(NLOS\ condition):&\ \hat{\sigma}_{nlos} \geq \varepsilon\sigma_m \end{aligned} \tag{6}$$

The parameter ε can be chosen according to an environment. The section of measured distance data $d_m(t_k)$ from each receiver can be determined whether NLOS noise is included or not by using this hypothesis test. In order to estimate the accurate position of an emitter, the identified LOS distance data is used for geolocation.

III. Geolocation Formula Using AOA Measurement

Among many kinds of geolocation approaches, AOA method is one of the most efficient ones. Using the measured angle data from each receiver, we can acquire the geolocation formula. The geolocation equation can be derived if the number of available AOA measurement is more than two [11]. However, the intersection point of AOA lines is not just only one since the measurement error causes that the lines of bearing are wrong drawn. Also, we formulate the objective function which means the error amount of position estimation in this section. The measured angle of the propagation signal from the emitter to the *i*-th receiver, denoted by φ_i, can be formulated with the unknown emitter position $\mathbf{o} = [x\quad y]^T$ and the known receiver position $\mathbf{p}_i = [x_i\quad y_i]^T$ as follows

$$\tan(\varphi_i) = \frac{\sin(\varphi_i)}{\cos(\varphi_i)} = \frac{y - y_i}{x - x_i} \tag{7}$$

In a real environment, the AOA variable φ_i includes the measurement error. Equation (7) can be rewritten by rearranging as the following equation

$$x\sin(\varphi_i) - y\cos(\varphi_i) = x_i\sin(\varphi_i) - y_i\cos(\varphi_i) \tag{8}$$

The matrix form of geolocation formula can be obtained through (8)

$$\mathbf{Ao} = \mathbf{b} \tag{9}$$

where

$$\mathbf{A} = \begin{bmatrix} \sin(\varphi_1) & -\cos(\varphi_1) \\ \vdots & \vdots \\ \sin(\varphi_M) & -\cos(\varphi_M) \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} x_1\sin(\varphi_1) - y_1\cos(\varphi_1) \\ \vdots \\ x_M\sin(\varphi_M) - y_M\cos(\varphi_M) \end{bmatrix}$$

The vector $\mathbf{o}$ is the solution of AOA based geolocation. The matrices $\mathbf{A}$ and $\mathbf{b}$ contain the measurement error since the AOA parameter φ_i is included. In order to solve this AOA geolocation problem denoted by (9), we suggest RWLS algorithm.

In order to obtain the optimized solution of an emitter position, the difference between the estimate position and real position, denoted by left and right side of (9) respectively, needs to be minimized. The (9) is not consistent due to the error included variables $\mathbf{A}$ and $\mathbf{b}$. The objective function which expresses the estimation error can be acquired as follows

$$F(\hat{\mathbf{o}}) = (\mathbf{A}\hat{\mathbf{o}} - \mathbf{b})^T \mathbf{C}(\mathbf{A}\hat{\mathbf{o}} - \mathbf{b}) \tag{10}$$

where the variable vector $\hat{\mathbf{o}} = [\hat{x} \quad \hat{y}]$ is the estimated solution. The parameter $\mathbf{C} = E[\mathbf{e}\mathbf{e}^T]$ is the covariance matrix which gives the weight to AOA measurement data. The variable $\mathbf{e}$ is the estimation error. In order to obtain the optimized solution of the objective function which is denoted by (10), the RWLS method applies to $F(\hat{\mathbf{o}})$.

IV. Optimized Solution Using RWLS

In order to apply the RWLS algorithm to the objective function using AOA method, we need to obtain the geolocation solution with respect to the Weighted Least Square (WLS) method. The WLS scheme is an extended case of an LS algorithm with covariance matrix which denotes the importance of the measurement data [12]. Compared with the WLS algorithm, the recursive process of the RWLS method provides fast computational rate and handle more additional measurement data. The general WLS solution of (10) in Section III is expressed as follows.

$$\mathbf{o}_{\mathbf{WLS}} = (\mathbf{A}^T\mathbf{C}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{C}\mathbf{b} \tag{11}$$

where $\mathbf{o}_{WLS}$ is an estimated position of an emitter through the WLS method. In order to apply the RWLS approach to objective function, the geolocation formula, denoted by (9), needs to be rewritten in a different formulation. Equation (9) can be rewritten as follows

$$\begin{bmatrix} \overline{\mathbf{A}}_k \\ \mathbf{A}_{k+1} \end{bmatrix} \mathbf{o} = \begin{bmatrix} \mathbf{b}_k \\ b_{k+1} \end{bmatrix} \tag{12}$$

where $\overline{\mathbf{A}}_k = [\mathbf{A}_1 \ \mathbf{A}_2 \cdots \mathbf{A}_k]^T$ expresses the matrix which includes the k data. The vector $\mathbf{b}_k$ represents $[b_1 \ b_2 \cdots b_k]^T$. $\mathbf{A}_{k+1}$ and b_{k+1} mean ($k+1$)-th AOA raw vector and scalar parameter, respectively. The WLS solution $\hat{\mathbf{o}}_{k+1}$ denoted by (11) can be rewritten in the same way as

$$\hat{\mathbf{o}}_{k+1} = (\overline{\mathbf{A}}_{k+1}{}^T\overline{\mathbf{C}}_{k+1}\overline{\mathbf{A}}_{k+1})^{-1}\overline{\mathbf{A}}_{k+1}{}^T\overline{\mathbf{C}}_{k+1}\overline{\mathbf{b}}_{k+1}$$
$$\overline{\mathbf{C}}_{k+1} = \begin{bmatrix} \overline{\mathbf{C}}_k & 0 \\ 0 & \mathbf{C}_{k+1} \end{bmatrix} \tag{13}$$

With the definition of $\mathbf{Z}_{k+1} = (\overline{\mathbf{A}}_{k+1}^T\overline{\mathbf{C}}_{k+1}\overline{\mathbf{A}}_{k+1})^{-1}$ for notational simplicity, the following equation can be derived as

$$\overline{\mathbf{Z}}_{k+1}^{-1} = \overline{\mathbf{Z}}_k^{-1} + \mathbf{A}_{k+1}^T\mathbf{C}_{k+1}\mathbf{A}_{k+1} \tag{14}$$

Using (14), WLS solution can be rewritten as follows

$$\hat{\mathbf{o}}_{k+1} = \overline{\mathbf{Z}}_{k+1}[\overline{\mathbf{Z}}_k^{-1}\overline{\mathbf{Z}}_k\overline{\mathbf{A}}_k^T\overline{\mathbf{C}}_k\overline{\mathbf{b}}_k + \mathbf{A}_{k+1}^T\mathbf{C}_{k+1}b_{k+1}] \tag{15}$$

Since $\hat{\mathbf{o}}_k = \overline{\mathbf{Z}}_k\overline{\mathbf{A}}_k\overline{\mathbf{C}}_k\overline{\mathbf{b}}_k$, equation (15) can be rewritten as

$$\hat{\mathbf{o}}_{k+1} = \hat{\mathbf{o}}_k + \overline{\mathbf{Z}}_{k+1}\mathbf{A}_{k+1}^T\mathbf{C}_{k+1}(b_{k+1} - \mathbf{A}_{k+1}\hat{\mathbf{o}}_k) \tag{16}$$

Since $\overline{\mathbf{Z}}_{k+1}^{-1} = (\overline{\mathbf{A}}_k^T\overline{\mathbf{C}}_k\overline{\mathbf{A}}_k + \mathbf{A}_{k+1}^T\mathbf{C}_{k+1}\mathbf{A}_{k+1})^{-1}$, the recursive solution with RWLS algorithm can be derived as follows.

$$\hat{\mathbf{o}}_{k+1} = \hat{\mathbf{o}}_k + (\overline{\mathbf{A}}_k^T\overline{\mathbf{C}}_k\overline{\mathbf{A}}_k + \mathbf{A}_{k+1}^T\mathbf{C}_{k+1}\mathbf{A}_{k+1})^{-1} \times \mathbf{A}_{k+1}^T\mathbf{C}_{k+1}(b_{k+1} - \mathbf{A}_{k+1}\hat{\mathbf{o}}_k) \tag{17}$$

Equation (17) is the RWLS solution of a geolocation problem using AOA measurement data. The emitter position can be estimated with computed values using k

measured data ($\hat{\mathbf{o}}_k$, $\overline{\mathbf{A}}_k$, and $\overline{\mathbf{C}}_k$) and $(k+1)$ -th measurements ($\mathbf{A}_{k+1}$, $\mathbf{C}_{k+1}$, and b_{k+1}). We can derive the position of an emitter with relatively fast rate through (17) when we receive additional AOA data sets from any receiver. Also, more precise estimate position can be acquired with the covariance matrices $\overline{\mathbf{C}}_k$ and $\mathbf{C}_{k+1}$.

V. Simulation Results

Some simulations confirm the effectiveness of our proposed RWLS scheme based geolocation approach using AOA measurements in this section. In our simulation, the AOA data is measured by five receivers of which the positions are (0, 0), (30, 10), (100, 0), (0, 105), and (55, 100) km, respectively. In a real environment, each AOA signal contains measurement errors. The measurement errors are assumed to follow the Gaussian distribution. Also, we assume the signal propagation speed c is 1 for simplifying the computation.

In Fig. 1, the estimation trajectory comparison with our proposed RWLS approach and general LS algorithm is demonstrated. The thick dotted line means the estimation trajectory through RWLS and the thin dotted line is the general LS algorithm based estimation. The solid line means the true trajectory of an emitter. As shown in this figure, the estimated trajectory of an emitter using RWLS scheme is much closer to the true trajectory than that using general LS algorithm.

Fig. 2 shows the root mean square error (RMSE) comparison of RWLS and LS based estimation. The solid line and dotted line are RMSE using RWLS and LS algorithm at each sampling time, respectively. The RMSE of the estimation trajectory determined by the proposed approach is much less than that of the LS algorithm based estimation at almost every sampling time. In Fig. 3, RMSE of estimated emitter position is shown with the iteration process when the true emitter position is (55, 50) km. The RMSE of estimation converges to a minimized error from derived final solution which is computed with objective function through RWLS approach.

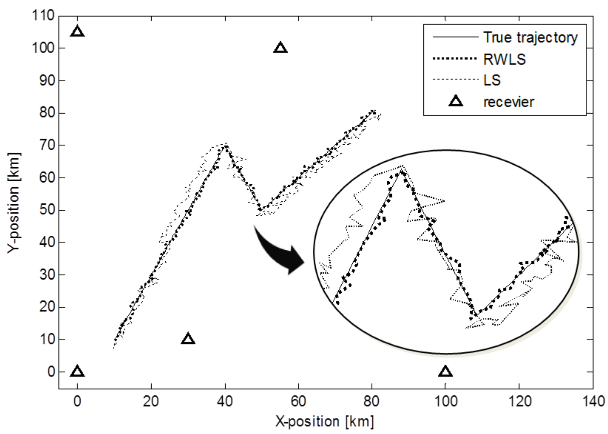

Figure 1. Estimated trajectory of emitter using RWLS method.

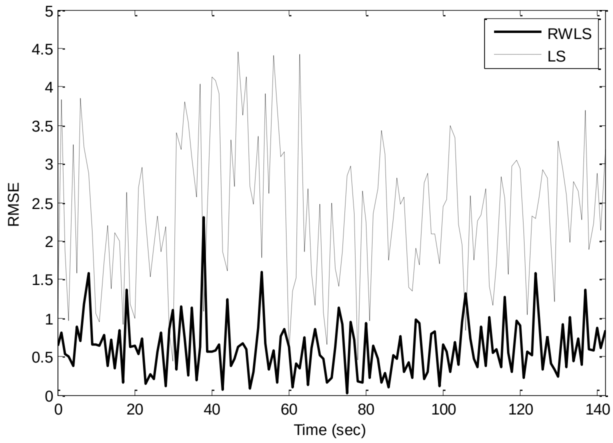

Figure 2. RMSE comparison of RWLS and general LS based estimation.

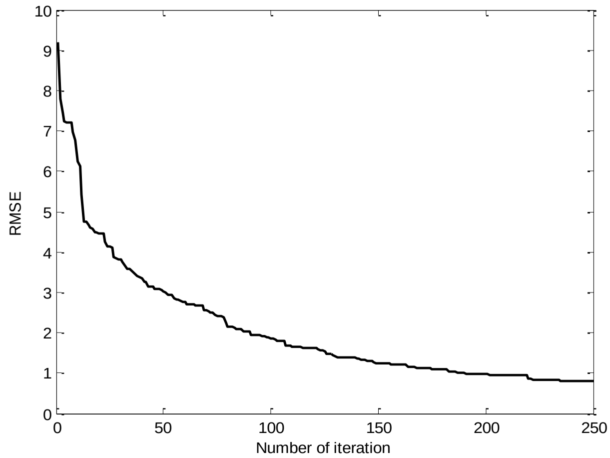

Figure 3. RMSE change depending on number of iteration increase.

VI. Conclusion

This paper presents the RWLS approach for geolocation using the AOA measurement data. We formulate the geolocation equation and objective function with measured AOA data in order to express an emitter position. Recursive solution is derived with application of RWLS algorithm to the formulated objective function. Our proposed scheme can be applied for position estimation whether an emitter moves or is stationary. Also, the covariance matrix which weights the AOA measurements contributes to acquire precisely estimated position of an emitter. Moreover, the proposed approach provides relatively rapid calculation speed for estimation when additional AOA data is received. Using our proposed RWLS scheme, more additional data can be handled easily for computation.

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