Feedback Control Design for VCM

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***Abstract*—A design of PD feedback control law for a class of the second order system to satisfy the desired performance requirements is presented. It is achieved by using root-locus approach. The desired specifications of the step-input system response are first transformed into a required region for the poles of the PD control closed-loop system. Ranges of the corresponding PD control gains are then derived to guarantee the poles of the closed-loop system lie within the targeted region. Besides, the proposed control law is also applied to the feedback control of Voice Coil Motor (VCM) to support the function of the so-called "Optical Image Stabilization (OIS)." Numerical results by using a referenced VCM model have demonstrated the success of the proposed design.**

***Index Terms*—PD control, VCM, Linear system**

I. Introduction

Recently, the technology of Optical Image Stabilizer (OIS) has been widely applied to the compensation of the unavoidable vibration caused by hand shaking while taking image or video so that the image or video will be less blurred. To achieve such a function, Chiu, Chao and Wu [1] proposed an algorithm to optimize OIS design by controlling the motion of the Voice Coil Motors (VCM) opposite to the vibration. For having better performance of motion control of VCM, the OIS system model including VCM might be needed. In 2011, Chao et al. proposed a mathematical model of an OIS system with 2-degree-of-freedom [2]. Wang et al. developed nonlinear dynamical equations for VCM in 2017 [3]. In addition, the system model is proposed in [4] to consider the effect of VCMs' copper loss. Besides, a lead-lag compensator is proposed to fulfill the system requirements of VCM [3]. That result is obtained by using the system model from the Bode plots of the experimental data. Instead of system compensation design from the frequency response, in this study we consider a different approach to simplify the design of controller.

It is known (e.g., [5]-[9]) that root-locus method is a very useful tool for the dynamical analysis and control of linear systems. Among the rules of the root-locus method, Break-Away and Break-In (BABI) condition is usually solved by using graphical approach [5], [6] or numerical calculation [7]. In our previous study of [10], we have re-derived the BABI condition for one-parameter control system by using different approach. Such a condition is also extended to the study of two-parameter linear control systems. In this paper, we will propose a standard PD control law by using the results of [10] for a class of second order linear system to meet the design specifications of timing response. Those results will then be applied to the compensation control of VCM.

The organization of the paper is as follows. First, the BABI condition used in root-locus approach is recalled from [10]. A design of PD control law for a class of the second order linear systems is then proposed to make the system to fulfill the design specifications. It is followed by the application to the control of a VCM. Numerical simulations are also obtained to demonstrate the success of the proposed design. Finally, a concluding remark is given.

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II. Preliminaries

Consider a typical unity feedback control system as shown in Fig. 1, where $G_C(s)$ is the transfer function of the controller and $G_P(s)$ is the transfer function of the system plant. Besides, the system input and output are denoted as $r(t)$ and $c(t)$, respectively.

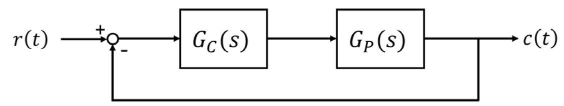

Figure 1. Block diagram of unity negative feedback control system.

The so-called "Break-Away/Break-In (BABI) condition" for the controlled system depicted in Fig. 1 is recalled from (e.g., [8], [10]) as stated below.

Lemma 1. (e.g., [8], [10]) Suppose $G_P(s)=q(s)/p(s)$ and $G_C(s)=K$ is a real constant. The root-locus of the closed-loop system defined in Fig. 1 will break away from or break into the real axis at $s=s^*$ with respect to the value of K if the following two conditions hold: (i) $K=-1/G_P(s^*)$ and (ii) $p(s)q'(s)-p'(s)q(s)\big|_{s=s^*}=0$.

Note that, the notation $(\ )'$ denotes the first derivative with respect to s and will be used in the sequel.

The result of the BABI condition has been extended to the study of control system with a non-constant gain of $G_C(s)$ in [10] as given below.

Lemma 2. [10] Suppose $G_P(s)=q(s)/p(s)$ and $G_C(s)=K_1\cdot\left[n_1(s)+K_2\cdot n_2(s)\right]/d(s)$. Then the root-locus

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of the closed-loop system shown in Fig. 1 will break away from or break into the real axis at $s = s^*$ if the following two conditions hold:

(i) $K_1 = \dfrac{-d(s^*)p(s^*)}{q(s^*)\left[n_1(s^*) + K_2 \cdot n_2(s^*)\right]}$ and

(ii) $T_1(s^*) + K_2 \cdot T_2(s^*) = 0$, with

$$T_i(s^*) = \{d(s)p(s)[n_i(s)q(s)]' - n_i(s)q(s)[d(s)p(s)]'\}|_{s=s^*}$$

for $i = 1$ and 2.

The results of Lemma 2 will be applied to the control law design for the system shown in Fig. 1 to satisfy a given performance requirement. Details are given below.

III. Control Law Design

Consider a class of the second order system as depicted in Fig. 1 with

$$G_P(s) = \frac{k}{s^2 + as + b}. \tag{1}$$

Here, we assume the open-loop system $G_P(s)$ as a pair of complex conjugate poles p_1 and p_2 with $a,\ b > 0$ while $k > 0$ denotes the system gain.

The major goal of this section is to design a controller $G_C(s)$ so that the closed-loop system given in Fig. 1 will satisfy the desired performance requirement.

It is known that the timing response of a linear system can be determined by the so-called "dominant poles." Here, we consider the dominant poles are a pair of complex conjugates and the desired performance specifications for this study are the settling time and the maximum overshoot for step-input timing response.

Denote $G^*(s)$ as the transfer function for the second order system which will meet the desired system specifications. Let

$$G^*(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{2}$$

where ζ is the damping ratio and ω_n is the undamped natural frequency. The system output $y(t)$ of system (2) for the unit-step input response can be expressed as (for $t \geq 0$)

$$y(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin\left(\omega_n\sqrt{1-\zeta^2}\,t + \cos^{-1}\zeta\right) \tag{3}$$

In general, the settling time and the maximum overshoot for the unit-step input timing response are selected as performance requirements (e.g., [8]) for control system design. The settling time of the system is known to be related to the value of $\zeta\omega_n$ no matter what it is defined in 5% [8], 2% [11] or 1% of the error with respect to the final value. Thus, in this study we will take T_{settle} as a term to play the role of the required settling time. We can then have the desired system performance as given in the following design specifications.

A. Design Specifications

$$\text{(i)}\ \zeta\omega_n \geq 1/T_{settle} \tag{4}$$

$$\text{(ii)}\ M_p\% = \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) \leq M_p^*\%, \tag{5}$$

where $M_p\%$ is the percentage of the overshoot.

It is noted that the values of T_{settle} and $M_p^*\%$ will be determined by the designer to fulfill the system requirements. In addition, the condition (ii) of Design Specifications listed above can also be rewritten as the following specification:

$$\zeta \geq \frac{-\ln\left(M_p^*\%\right)}{\sqrt{\pi^2 + [\ln\left(M_p^*\%\right)]^2}} = \zeta^*, \text{ or} \tag{6}$$

$$\theta \leq \cos^{-1}\left(\zeta^*\right) = \theta^* \tag{7}$$

Note that, the operator "ln" in (6) above denotes the natural log and the values of θ^* and T_{settle} become the required specifications for the control design as depicted in the shaded region of Fig. 2.

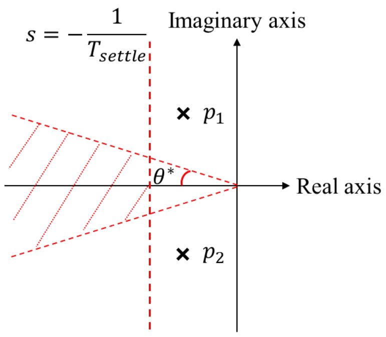

Figure 2. The locations of the open-loop system poles p_1 and p_2 and the desired performance region.

In this study, we assume the open-loop system do not meet the system requirements as depicted in Fig. 2. So, the next step is to construct a suitable controller $G_C(s)$ such that the closed-loop system can satisfy the desired performance requirements. Here, we consider three simple types of controller such as P, PI, or PD controllers.

First, we consider the possibility of having P controller, i.e., $G_C(s) = K_P$ to fulfill the requirements. As depicted in Fig. 3, the root-locus of the P control closed-loop system fails to pass through the desired performance region.

Next, we consider the case of the system with PI type controller, i.e., $G_C(s)=K_P(s+K_I/K_P)/s$ to fulfill the requirements. Unfortunately, it is clear from Fig. 4 that the root-locus of the PI control closed-loop system will fail to pass through the desired performance region either.

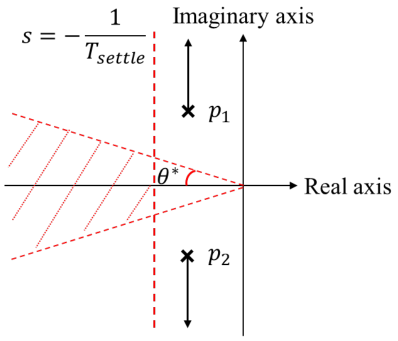

Figure 3. Root locus for the closed-loop system with P controller.

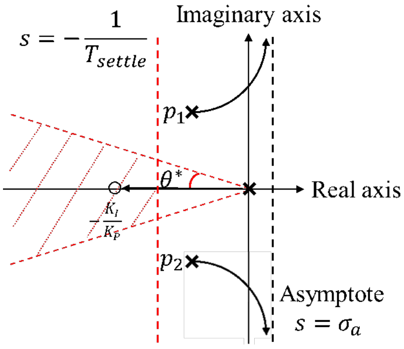

Figure 4. Root locus for the closed-loop system with PI controller.

Now, we consider the third type of controller. That is, to choose PD type controller with $G_C(s)=K_D\cdot(s+K_P/K_D)$. A typical root-locus of the PD control closed-loop system is given in Fig. 5, which demonstrates the possibility of meeting the design requirements. Thus, in the following we will focus on the design of PD controller.

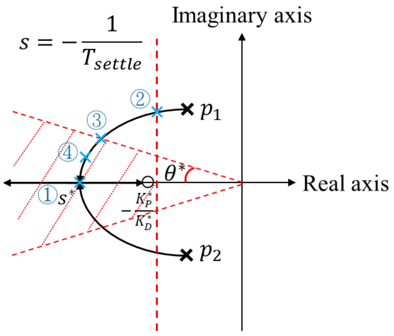

Figure 5. Root locus for the closed-loop system with PD controller.

In the previous study of [10], we have re-derived the Break-Away and Break-In (BABI) condition used in root-locus method of one-parameter control system by using different approach. Such a condition is also extended to the study of two-parameter linear control systems recalled as Lemma 2 above. In the following, we will apply Lemma 2 to construct a PD type control law for the second order linear system as defined in (1) to meet the design specifications listed in (4)-(5). Compared with the conventional design by using root-contours, the proposed approach can give a guideline for choosing control gains instead of those obtained by trial-and-error from the root-contours scheme. Details are given below.

As stated in Lemmas 1 and 2 above, we can obtain the location of the so-called "BABI" condition on the s-plane by giving the structure and/or numerical values of the proposed controller $G_C(s)$. In this paper, we consider a different approach by choosing the BABI location as the desired objective and find the condition and/or the numerical values of the proposed controller. It is clear from Fig. 2 that we need to have the poles of the closed-loop system lying on the shaded region. So, we can pick the desired break-in location of the root-locus on the left-hand side of the vertical line $s=-1/T_{settle}$ and push the root trajectory to enter the shaded region.

In the following design, based on the analysis above we will consider the controller as a PD type controller, i.e., $G_C(s)=K_D(s+K_P/K_D)$. That mean, the closed-loop system will have an extra zero at $s=-K_P/K_D$. According to the root-locus method, the root trajectories will go to the new extra zero and $s=-\infty$ if we take K_D as a varied control gain with $K_D\to+\infty$.

According to Lemma 2, we can rewrite the controller as follows:

$$G_C(s)=sK_D+K_P=K_1\left[n_1(s)+K_2\cdot n_2(s)\right]/d(s),$$

where $d(s)=1$, $K_1=K_D$, $n_1(s)=s$, $K_2=\dfrac{K_P}{K_D}$, and $n_2(s)=1$.

Next result follows readily from Lemma 2.

Theorem 1. Let $s=s^*$ be the desired break-in location of the root-locus for the system (1) with $G_C(s)=K_D\cdot(s+K_P/K_D)$. Then the control gains $K_P=K_P^*$ and $K_D=K_D^*$ to support the break-in condition with $K_D^*=-\dfrac{2s^*+a}{k}$ and $K_P^*=\dfrac{(s^*)^2-b}{k}$.

Remark 1. As stated in Theorem 1, the extra zero contributed by the PD controller should be selected as $z=K_P^*/K_D^*$ and treat the control gain K_D as a varied control gain for the compensation design.

Now, we seek for the range of the varied control gain K_D which will guarantee the poles of the closed-loop system lying within the desired shaded region. It is observed from Fig. 5 that the root-locus of the closed-loop system might first pass through the constraint on the vertical line $s=-1/T_{settle}$ at the location ② and then pass through the other constraint at the location ③ or vice versus. For the case of the location ②, we can obtain the condition of K_D as $K_D = \frac{2\zeta\omega_n - a}{k}$. In addition, the condition for the case of the location ③ is solved as $K_D = \left(-a + 2\zeta z + \sqrt{(\zeta z)^2 + \zeta b - \zeta^2 az}\right)/k$. Here, we have $z = K_P^* / K_D^*$. Therefore, the minimum bound for K_D driving the poles of the closed-loop system to lie within the desired shaded region can be determined as

$$K_{D\min} = \max\left\{\frac{2\zeta\omega_n - a}{k}, \frac{-a + 2\zeta z + \sqrt{(\zeta z)^2 + \zeta b - \zeta^2 az}}{k}\right\}.$$

Then we have the next result.

Theorem 2. Let $s = s^*$ be the desired break-in location of the root-locus for the system (1) with $G_C(s) = K_D(s+z)$ and $z = K_P^* / K_D^*$. Then the poles of the closed-loop system will lie within the desired shaded region if $K_{D\min} \le K_D < K_D^*$.

From the results of Theorems 1 and 2, we can then summarize the design procedure as given in Algorithm 1 below.

Algorithm 1.

Step 1. Construct the desired region shown as in Fig. 5 to meet the desired specification as given in (4)-(7).

Step 2. Choose the BABI point $s = s^*$ satisfied conditions $s^* < -1/T_{settle}$ and solve for the value of K_D^* and the extra zero $z = K_P^* / K_D^*$.

Step 3. Solve for the value of $K_{D\min}$ and choose the control gain K_D to satisfy the constraint: $K_{D\min} \le K_D < K_D^*$.

Remark 2. In the proposed design above, we have focused on the design of pole placement. In order to fit the form of the desired system as given in (2), a pre-controller is needed to modify the numerator part of the designed closed-loop system.

IV. Application to the Control of VCM

In the following, we will apply the proposed PD control law proposed in the previous section to the motion control of VCM. First, we will try to obtain the OIS model from [3]. The procedure of Algorithm 1 will then be used to design the control law.

In the study of [3], an OIS model is identified by using experimental frequency responses. From the Bode plots obtained in [3], the model of the open-loop system of the testing VCM can be concluded as a second order system with a pair of complex conjugate poles. In this study, we have tried to estimate the corresponding system model as given by

$$G(s) = \frac{23507}{s^2 + 51s + 13278.25}, \tag{8}$$

which has two complex conjugate poles at $-25.5 \pm j363.5$ with system gain $k = 23507$. The Bode plot for the system (8) is obtained and given in Fig. 6, which is found to be very close to the one in [3]. Next, we choose the settling time $T_s = 0.0104$ second and the maximum overshoot $M_p^*\% = 8\%$ for the required design specifications. Refer to the calculation of system time constant meeting the required settling time of [11], we choose $T_{settle} = T_s / 4$ as the bound for system time constant $\zeta \cdot \omega_n$.

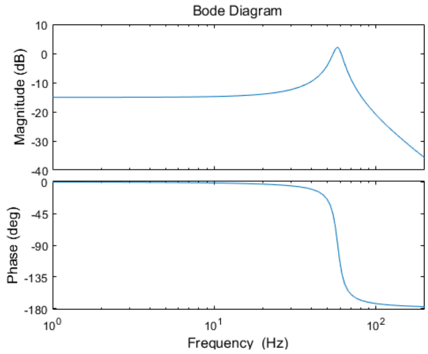

Figure 6. The Bode plot of the estimated system model (8).

We then have the design specifications as plotted in the shaded area of Fig. 5. According to Algorithm 1, for the controller design we need to choose the desired break-in location for BABI point $s^* < -1/T_{settle}$, i.e., $s^* < -384.6$.

In this study, we choose $s^* = -500$ for the design. From Theorem 2, we then have $K_D^* = 0.0404$, $K_P^* = 4.9865$ and $K_{D\min} = 0.0306$. The extra zero is $z = 123.5$. The range of K_D for meeting the design specifications is calculated as $0.0306 \le K_D < 0.0404$. The root-locus of the designed system is obtained as depicted in Fig. 7. Choose the designed values to be $K_D = 0.031$ and $K_P = K_D \cdot z = 3.829$. We then have the corresponding unit-step input response as shown in Fig. 8. It is found that the maximum overshoot is about 1% and the settling time $T_s = 0.0084$ which meet the desired system specifications.

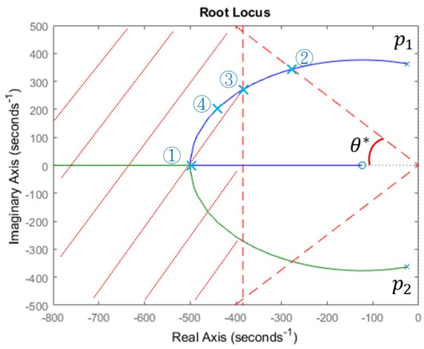

Figure 7. The root-locus of system with PD control having $s^* = -500$.

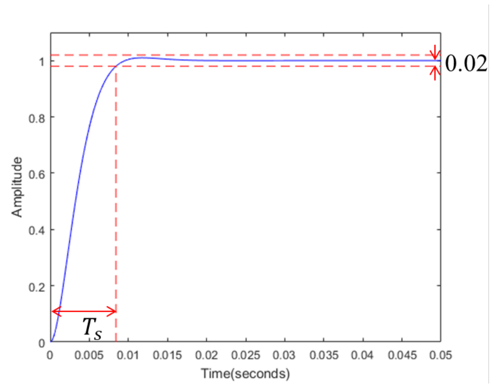

Figure 8. Step response of system with PD control for $K_D = 0.031$ and $K_P = 3.829$ for the case of $z = 123.5$.

V. Conclusions

In this study, we have proposed a PD type controller for a class of second order linear systems to meet the designed specifications. The proposed control law is also applied to the control design for VCM, which has demonstrated the success of the design. In this paper, we only consider the system possessing a pair of complex conjugate poles. The proposed results will not be difficult to be extended for the design of the system with two real poles.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

The main contribution of the first author is to provide mathematical analysis, while the major tasks for both of the second and third authors are doing the numerical simulations and word processing of the manuscript.

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