

# The 3D Passive Position Research for Three-Element Array in 0~359°

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**Abstract**—Three-element array is often used for measuring the distance between the object and the reference element. Cooperating with other devices or increasing the number of elements, the three-element array may achieve 3D (3D: three-dimensional) passive position. To the problem of 3D passive position of three-element array, this paper provides a method achieving three-dimensional passive position based on the underwater acoustic channel propagation characteristics in 0~359°. At the same time, the method can solve the y-axis fuzzy direction of the linear three-element array without prior knowledge. The numerical analysis and simulation show that this paper can well complete three-dimensional passive position, especially for the shallow water short-range target.

**Index Terms**—three-element array, underwater acoustic channel, three-dimensional passive position

## I. INTRODUCTION

Because the active location need transmitter signal equipment and is easy exposure, the passive position technology of underwater target becomes one of the hot issues in sonar technology. The typical passive position method is achieved by measuring TDD (TDD: The time delay difference) of differential elements receiving signals [1]-[7]. But three-element array is used for the two-dimensional position [1]-[7], and doesn't relate to the 3D passive position of targets, especially for the line three-element array [6]. In this paper, we can achieve 3D passive position of three-element array by the propagation characteristics of the underwater acoustic channel [8]-[13] and virtual element. The method can solve the fuzzy direction of the y-axis [14] comparing to the line three-element array without prior knowledge, and also reduces the installation difficulty of the line array. This paper achieves high-precision 3D passive position through the correlation method and interpolation

method [15], [16]. The TDD between virtual element and real element is distinguished by the expert system [17]. Numerical analysis and simulation show that the method can achieve 3D passive position in 0~359°, especially for shallow water short-range targets.

## II. NON-LINEAR THREE-ELEMENT ARRAY POSITION PRINCIPLE AND ALGORITHM

### A. Underwater Acoustic Channel Propagation Characteristics

Because of multi-path effect of underwater acoustic channel, the target radiating signals may contain the direct rays, the surface-reflected rays, the seabed-reflected rays, the surface seabed-reflected rays, and the seabed surface-reflected rays when they arrive the receiving elements [8]-[13]. This paper will give a method that can achieve 3D passive position of three-element array by the propagation characteristics of the underwater acoustic channel and virtual element. Fig. 1 is sound rays Fig that contains the direct rays, the surface-reflected rays, the seabed-reflected rays.

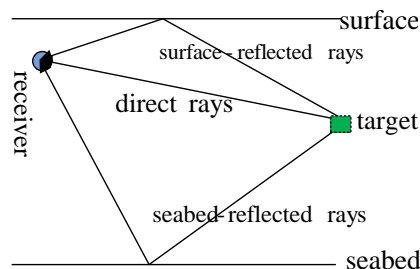


Figure 1. Sound rays Fig of underwater acoustic channels

According to Fig. 1, we can use the Eq. (1) denote the relationship between target radiating signals and receiver receiving signals.

$$x_j(t) = \sum_{i=1}^N A_{ij} \cdot s(t) \delta(t - t_{ij}), j = 1, 2, 3 \quad (1)$$

where,  $x_j(t)$  denotes the element  $j$  receiving signals,

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and  $A_j$  denotes the element  $j$  receiving the amplitude value of the ray  $i$ , and  $t_{ij}$  denotes the time delay of the element  $j$  receiving the ray  $i$ , and  $s(t)$  denotes the target radiating signals, and  $\delta(t-t_i)$  denotes the response function of the underwater acoustic channels, and  $N$  denotes the number of rays that arrive the receiver array elements through underwater acoustic channels.

**B. Non-linear Three-element Array Position Principle**

According to the section A, this paper provides a virtual element by the middle element through the sea surface. The virtual element receiving signal is the surface-reflected signal of the middle element. Fig. 2 is the three dimensional position schematic diagram of the three-element array based on underwater acoustic channel propagation characteristics.

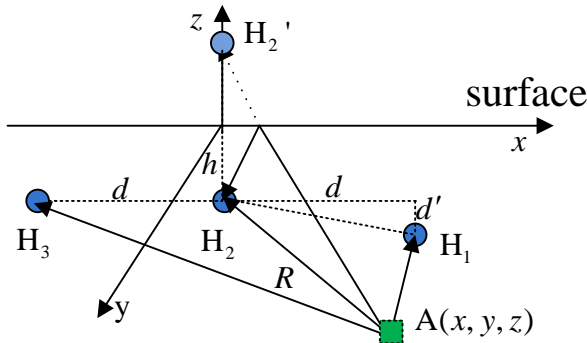


Figure 2. The 3D passive position schematic diagram of the three-element array in 0~359°

where,  $A(x, y, z)$  denotes the target, and  $H_1(d, d', h)$ ,  $H_2(0, 0, h)$ ,  $H_3(-d, 0, h)$  denote three receiving elements, and  $H_2'(0, 0, -h)$  denotes virtual element of  $H_2(0, 0, h)$ , and  $d, d', -h$  denote coordinate parameters, and  $R$  denotes the distance between  $A(x, y, z)$  and  $H_2(0, 0, h)$ . We can use (2.1)–(2.4) denote the distance between  $A(x, y, z)$  and  $H_2(0, 0, h)$ ,  $H_1(d, d', h)$ ,  $H_2'(0, 0, -h)$ ,  $H_3(-d, 0, h)$  in Eq.(2)

$$\begin{cases} x^2 + y^2 + (z-h)^2 = R^2, & (2.1) \\ (x-d)^2 + (y-d')^2 + (z-h)^2 = (R+c\cdot\tau_{21})^2, & (2.2) \\ (x+d)^2 + y^2 + (z-h)^2 = (R+c\cdot\tau_{23})^2, & (2.3) \\ x^2 + y^2 + (z+h)^2 = (R+c\cdot\tau_{22})^2, & (2.4) \end{cases} \quad (2)$$

where,  $\tau_{21}, \tau_{22}, \tau_{23}$  denote the time delay difference between  $H_1(d, d', h)$ ,  $H_2'(0, 0, -h)$ ,  $H_3(-d, 0, h)$  and  $H_2(0, 0, h)$ , and  $c$  denotes the sound speed. The part variables of Eq. (2) can be transformed as Eq. (3).

$$R_{21} = c \cdot \tau_{21}, R_{22} = c \cdot \tau_{22}, R_{23} = c \cdot \tau_{23} \quad (3)$$

According to the Eq. (2) and Eq. (3), we can get the coordinate value of x-axis, y-axis, z-axis as Eq. (4).

$$\begin{cases} x = \frac{R_{23}^2 + 2R \cdot R_{23} - d^2}{2d}, & (4.1) \\ y = \frac{2d^2 + d'^2 - R_{21}^2 - R_{23}^2 - 2R(R_{21} + R_{23})}{2d'}, & (4.2) \\ z = \frac{R_{22}^2 + 2R \cdot R_{22} - 4h^2}{4h}, & (4.3) \end{cases} \quad (4)$$

We can use (2.1), (4.1), (4.3), (4.3) solve the distance  $R$  as the Eq. (5), (6). when  $A_6 > 0$ :

$$R = \frac{\sqrt{A_1 - A_2 \times A_3 \times A_4 + A_5} + A_6}{A_6} \quad (5)$$

when  $A_6 < 0$ :

$$R = \frac{-\sqrt{A_1 - A_2 \times A_3 \times A_4 + A_5}}{A_6} \quad (6)$$

where,  $A_1, A_2, A_3, A_4, A_5, A_6$  are shown as Eq.(7). When  $R$  is known, we can use  $R$  and Eq. (4.1), (4.3), (4.3) solve the three-dimensional coordinates of the target.

$$\begin{cases} A_1 = \left( \frac{B_1 R_{23}}{d^2} + \frac{B_4 R_{23}}{4h^2} + \frac{B_2 B_3}{2d'^2} \right)^2 \\ A_2 = \frac{B_1^2}{d^2} + \frac{B_2^2}{d'^2} + \frac{B_4^2}{4h^2} \\ A_3 = \frac{B_3^2}{4d'^2} + \frac{R_{23}^2}{d^2} + \frac{R_{22}^2}{4h^2} - 1 \\ A_4 = d^2 h^2 d'^2 \\ A_5 = \frac{B_2 B_3 h^2 d'^2}{2} - \frac{B_4 R_{22} d^2 d'^2}{4} - B_1 R_{23} h^2 d'^2 \\ A_6 = \frac{B_3 d^2 h^2}{2} + \frac{d^2 R_{22} d'^2}{2} + 2h^2 d'^2 R_{23}^2 - 2d^2 h^2 d'^2 \end{cases} \quad (7)$$

where,  $B_1, B_2, B_3, B_4$  are shown as Eq. (8).

$$\begin{cases} B_1 = R_{23}^2 - d^2 \\ B_2 = -R_{23}^2 - R_{21}^2 + d'^2 + 2d^2 \\ B_3 = 2(R_{23} + R_{21}) \\ B_4 = R_{22}^2 - 4h^2 \end{cases} \quad (8)$$

Similarly, we can image the virtual element  $H_1', H_3'$  from the element  $H_1, H_3$  based on the sea surface. Then according to the Eq. (5) to (8), we can get  $A(x, y, z)$ .

We can calculate the differential coefficient for (4.1), (4.3), (4.3), then obtain the position error as the Eq. (9).

$$\begin{cases} \Delta x = \frac{(2(R_{23} + R)\Delta R_{23} + 2R_{23}\Delta R)d - (R_{23}^2 + 2R \cdot R_{23})\Delta d}{2d^2} - \frac{\Delta d}{2} \\ \Delta y = \frac{2d\Delta d - 2R_{23}\Delta R_{23} - 2R_{21}\Delta R_{21} - 2\Delta R(R_{21} + R_{23}) - 2R(\Delta R_{21} + \Delta R_{23})}{2d'} \\ + \frac{\Delta d'}{2} - \frac{(2d^2 - R_{21}^2 - R_{23}^2 - 2R(R_{21} + R_{23}))\Delta d'}{2d'^2} \\ \Delta z = \frac{(2R_{22}\Delta R_{22} + 2\Delta R \cdot R_{22})h - (R_{22}^2 + 2R \cdot R_{22})\Delta h}{4h^2} \end{cases} \quad (9)$$

### III. THE NUMERICAL ANALYSIS

In order to verify that the method can realize 3D passive position in  $0\sim 359^\circ$ , we use Fig. 3 as the simulated target track for numerical simulation.

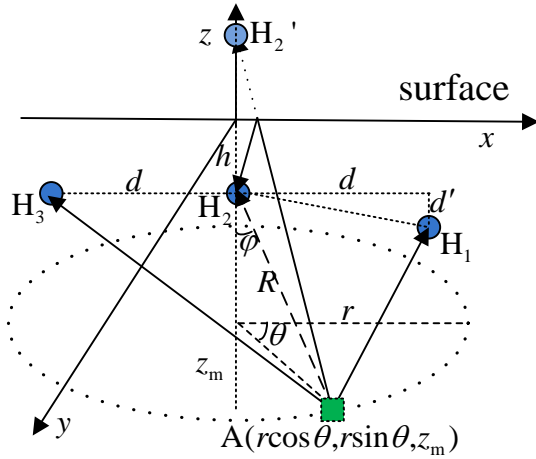


Figure 3. The figure of simulated target track

Where,  $r = R\sin\phi$  denotes the horizontal radius of the target, and  $z_m = R\cos\phi$  denotes depth of target, and  $R$  denotes the distance between target and  $H_2$ , and  $\phi$  denotes the pitch angle of target, and  $\theta$  denotes azimuth of target.

#### A. 3D Passive Position in $0\sim 359^\circ$

In order to simulate position effects of the actual target, in this paper, we use  $2\sim 5$  kHz white noise as target

radiating signal. Target radiating signal becomes the direct rays and the sea surface-reflected rays arriving the receiver. The amplitude of the direct rays and the sea surface-reflected rays is calculated by spherical wave propagation characteristics. The TDD between the elements is calculated by path difference between the target and each element. The signal-to-noise ratio is  $SNR = -6$  dB, and sound speed is  $c = 1500$  m/s, and the sample rate is  $f_s = 250$  kHz of target radiating signal, then the receiver's sample rate is  $f_s = 25$  kHz. So original TDD error is  $\Delta\tau = 2 \mu s$ , but the receiver's TDD error is  $\Delta\tau = 20 \mu s$  if only using correlation method. We can use correlation method and 10 times frequency domain interpolation get the high-precision TDD [15], [16].

$$\begin{cases} Cx_i x_j = E[x_i(t) \cdot x_j(t)] \\ FCx_i x_j = \text{fft}(Cx_i x_j) \quad , i=2; j=1,2,3 \\ \overline{Cx_i x_j} = \text{ifft}(FCx_i x_j, 10f_s) \end{cases} \quad (10)$$

Where,  $Cx_i x_j$  is the result of the correlation between  $i$  element receiving signal  $x_i(t)$  and  $j$  element receiving signal  $x_j(t)$ , and  $E[\cdot]$  is the mathematical expectation function, and  $\text{fft}(\cdot)$  is the Fourier transform function, and  $\text{ifft}(\cdot)$  is the inverse Fourier transform function. So the accuracy of  $\overline{Cx_i x_j}$  is increased by 10 times comparing the accuracy of  $Cx_i x_j$ . Through the expert system [17], we can obtain the time delay differences  $\tau_{21}, \tau_{23}, \tau_{22}$ .

In order to observe the details of the target position, Fig. 4 shows the result of 3D passive position by 100 times independent statistics when  $r = 500$  m.

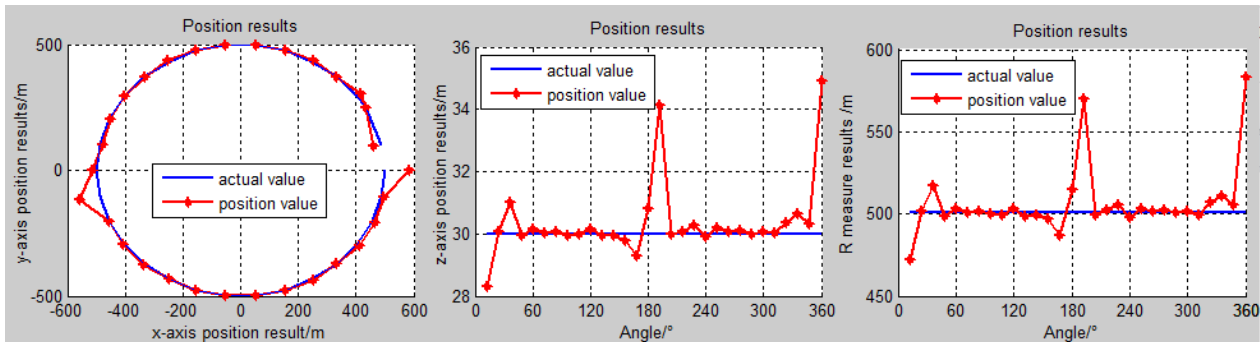


Figure 4. The position results when  $d = 15, h = 10, d' = 2, r = 500$  m.  $\theta = 30^\circ \sim 150^\circ, 210^\circ \sim 330^\circ$

According to Fig. 4, we can find that the method can achieve the 3D passive position in  $0\sim 359^\circ$ , because the projection aperture in the y-axis direction is relatively small, the position accuracy is not high in the x-axis direction. But we can use cross array solve the low position accuracy in the x-axis direction.

In order to show that the elements position has impact on 3D passive position results, we will give the numerical simulation results from the following four aspects.

- 1) Assuming the coordinate parameters are  $d = 10, h = 10, d' = 2$ , Fig. 5 is position errors.
- 2) Assuming the coordinate parameters are  $d = 15, h = 10, d' = 2$ , Fig. 6 is position errors.
- 3) Assuming the coordinate parameters are  $d = 15, h = 10, d' = 10$ , Fig. 7 is position errors.
- 4) Assuming the coordinate parameters are  $d = 15, h = 20, d' = 10$ , Fig. 8 is position errors.

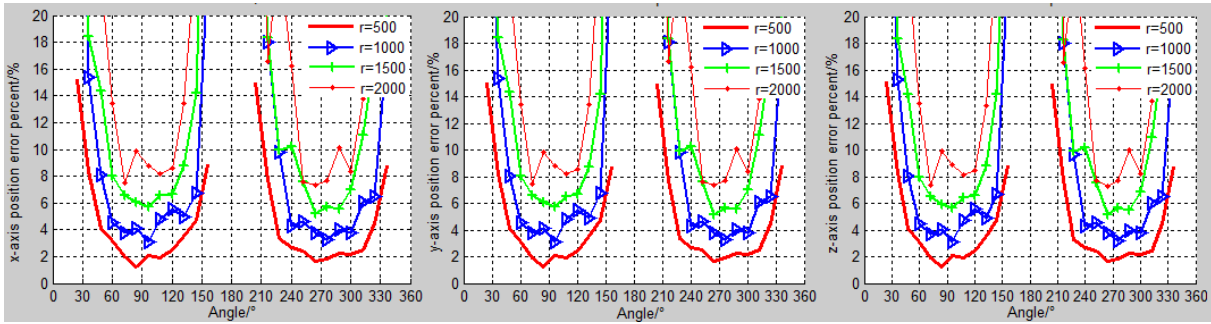


Figure 5. The position error percent when  $d = 10, h = 10, d' = 2, r = 500 : 500 : 2000$  m

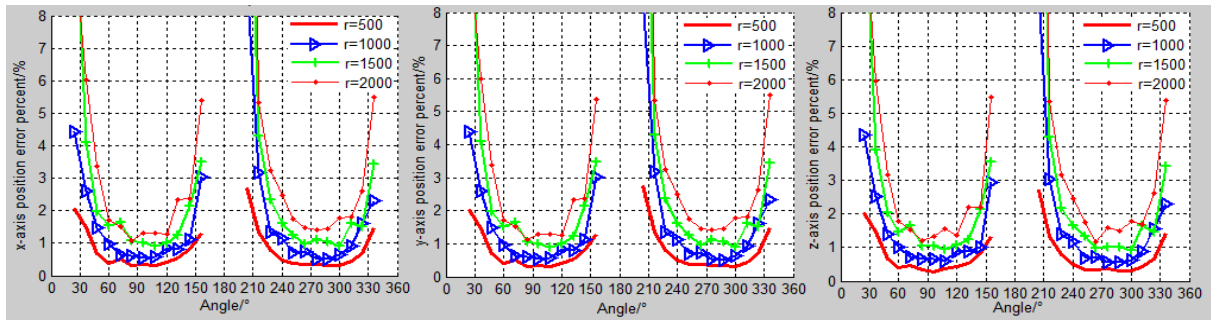


Figure 6. The position error percent when  $d = 15, h = 10, d' = 2, r = 500 : 500 : 2000$  m

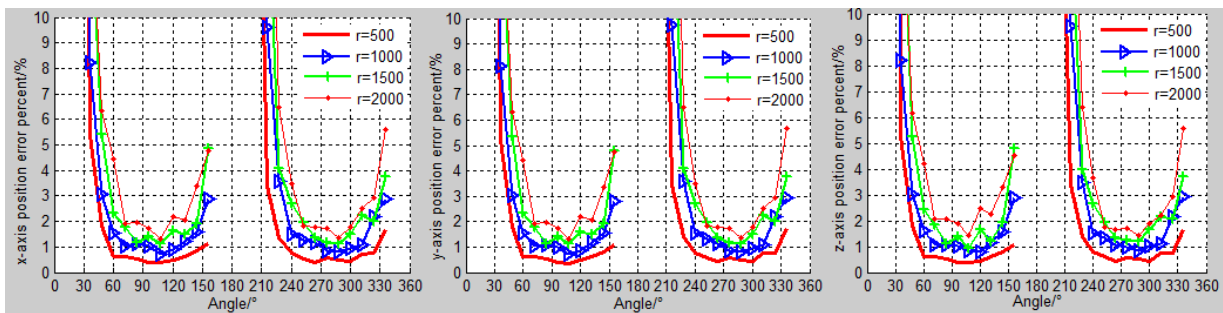


Figure 7. The position error percent when  $d = 15, h = 10, d' = 10, r = 500 : 500 : 2000$  m

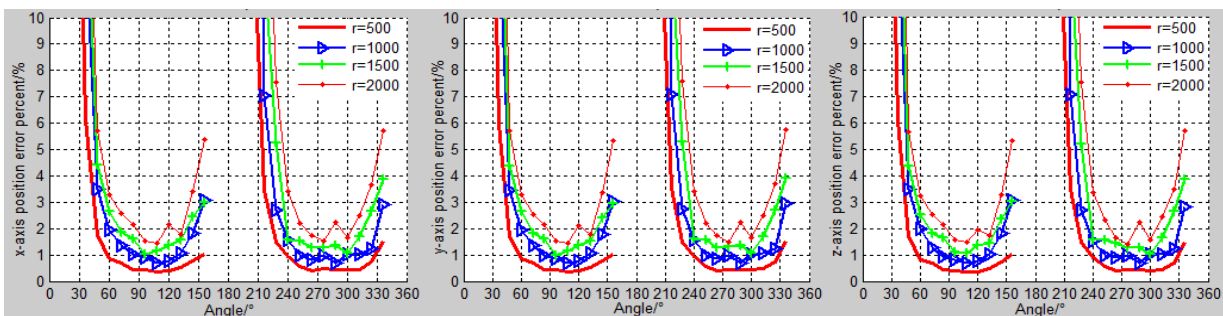


Figure 8. The position error percent when  $d = 15, h = 20, d' = 10, r = 500 : 500 : 2000$  m

According to the results of Fig. 5 — Fig. 8, we can get the following conclusions:

- 1) The position accuracy becomes worse as the distance  $R$  becomes larger. Especially, when  $d$  is small (comparing Fig. 5 with Fig. 6).
- 2) Because the element  $H_1$  is located in the fourth quadrant in the simulation, as  $d'$  becomes larger, the 2,4 quadrant position accuracy becomes better, and the 1,3 quadrant position accuracy becomes worse in the x-axis direction (comparing Fig. 6 with Fig. 7).

3) The parameter  $h$  has a small impact on the position accuracy (comparing Fig. 7 with Fig. 8).

4) The position accuracy of x-axis direction and y-axis direction becomes better as  $d$  becomes larger, especially for shallow water short-range targets (comparing Fig. 5 with Fig. 6).

The simulation results show that this method can achieve high-precision 3D passive position in  $0 \sim 359^\circ$  especially for shallow water short-range targets in  $30^\circ \sim 150^\circ$  and  $210^\circ \sim 330^\circ$ . At the same time, simulation

results prove that the parameter  $d$  in x-axis direction has a great impact on position accuracy, so high position accuracy need large  $d$ . According to spatial correlation radius of noise signal and convenient installation,  $d$  is not too large.

#### IV. CONCLUSION

This paper uses multi-path effect of underwater acoustic channel propagation characteristics and virtual element to obtain the 3D passive position formula of the non-linear three-element array, and solve the fuzzy direction of the y-axis without prior-knowledge. Numerical simulation has verified that this method can effectively achieve 3D passive position in  $0\sim 359^\circ$  especially for shallow water short-range targets in  $30^\circ\sim 150^\circ$  and  $210^\circ\sim 330^\circ$ . At the same time, we can fuse the redundant data to achieve better position. But to the problem of x-axis direction being not good in this paper, we can increase the element in y-axis to solve. We need further explore for three-element array achieving same precision in  $0\sim 359^\circ$ .

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