Improvement of the EMD-SG Denoising Method

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***Abstract*—The Empirical Mode Decomposition (EMD) is a new method for processing nonlinear and non-stationary signals. In this paper, an improvement of a denoising scheme using EMD was proposed. A noisy signal was decomposed adaptively into IMFs by using EMD method. The basic principle of the original method was to filter by Savitzky-Golay filter (SG) each IMF and then to reconstruct the signal from the filtered IMFs. The new method for reducing noise was based on multiple pass the noisy IMFs through a Savitzky-Golay filter two or more times. The EMD-SG multiple pass denoising technique was tested on four noisy simulated signals (Doppler, Blocks, Bumps and Heavysine) corrupted by white Gaussian noise with different Signal-to-Noise Ratio (*SNR*). The performance of proposed denoising method was quantitatively evaluated using *SNR* and Mean Square Error (*MSE*). The evaluation results showed that the proposed EMD-SG multiple pass performed better than the original EMD-SG method.**

***Index Terms*—EMD, denoising, Savitzky-Golay filter**

I. Introduction

The Empirical Mode Decomposition (EMD) method was introduced by Huang *et al.* [1] for the analysis of non stationary and/or nonlinear signals. The EMD is designed to adaptively decompose a signal into a sum of Intrinsic Mode Functions (IMFs) using a sifting process. The analysis of structured broadband stochastic processes such as fractional Gaussian noise by EMD showed that this method is organized in dyadic filter bank [2]. Consequently, many EMD-based denoisings were proposed to remove noises from observed data [3]-[5]. The filtering principle was based on the reconstruction of the signal with all IMFs previously thresholded, as inspired by standard wavelet thresholding, or filtered [3], [4]. Since the useful information of the signal is often concentrated on the low-frequency IMFs (higher order IMFs) and the noise is primarily localised on high-frequency IMFs (lower order IMFs). Therefore, the denoising method can be based on the partial construction of the signal using only the IMFs that contain useful information and rejecting those that carry noise [5].

In [3], [4], Boudraa *et al.* proposed a signal denoising scheme with each pre-filtered IMFs by Savitzky-Golay filter to estimate the signal (EMD-SG). This EMG-SG denoising method was tested on simulated and real with different noise levels and compared to Wavelets, Averaging and Median denoising methods. The results showed the EMD-SG outperform EMD-Soft, the Averaging and the Median methods for the five test signals. The Wavelets method performs better *SNR* than the EMD-SG of 1.4 dB for only Doppler signal. Indeed, Savitzky-Golay filter or polynomial least squares filter is widely-known as a low-pass filter and is well-adapted for data smoothing [6].

In this paper, a new denoising method based on EMD and multiple pass of the all IMFs through a Savitzky-Golay filter two or more times was proposed. Thus, the cleaned signal was reconstructed using filtered IMFs.The effectiveness of the proposed method was tested on four simulated signals with different *SNR* and compared to the one based in simple EMD-SG filter. The obtained results showed the better performance of the new method compared to the EMD- SG previously used for denoising. The results were quantitatively evaluated using *SNR* and mean *MSE*.

II. EMD Algorithm

EMD is an adaptive method to decompose asignal $x(t)$ into a series of IMFs. The IMFs must satisfy the following two conditions:

(1) The number of maximum must equal the number of zeros or differ at most by one.

(2) In each period, it is necessary that the signal average is zero.

The EMD algorithm (the sifting process of extracting IMFs) consists of the following steps [1]:

1. Find local maxima and minima in $x(t)$ to construct the upper and lower envelopes respectively using cubic spline interpolation.

2. Calculate the mean envelope, $m(t)$, by averaging the upper and lower envelopes.

3. Calculate the temporary local oscillation $h(t) = x(t) - m(t)$.

4. Calculate the average of $h(t)$, if average $h(t)$ is close to zero, then $h(t)$ is considered as the first IMF, named $c_i(t)$, otherwise, repeat steps 1-3 while using $h(t)$ for $x(t)$.

5. Calculate the residue $r(t) = x(t) - c_i(t)$.

6. Repeat steps from (1) to (5) using $r(t)$ for $x(t)$ to obtain the next IMF and residue.

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The decomposition process stops when the residue $r(t)$ becomes a monotonic function or a constant that no longer satisfies the conditions of an IMF.

$$x(t) = \sum_{i=1}^{N} c_i(t) + r_N(t) \tag{1}$$

The sifting process is continued until the last residual is either a monotonic function or a constant.

III. EMD Based Denoising

A. Principle

Having a noisy signal $y(t)$ given by:

$$y(t) = x(t) + \eta(t) \tag{2}$$

where $x(t)$ is the noiseless signal and $\eta(t)$ is independent noise of finite amplitude.

In EMD-based denoising methods, the noisy signal $y(t)$ is first decomposed into noisy IMFs: $c\eta_i(t)$. These noisy IMFs can then be filtered in order to obtain $\hat{c}_i(t)$ that is an estimation of the IMFs of the noiseless signal. Therefore, the denoising signal $\hat{x}(t)$ can be obtained as follows:

$$\hat{x}(t) = \sum_{i=1}^{N} \hat{c}_i(t) + r_N(t) \tag{3}$$

Several EMD-based denoising methods have been proposed [3]-[5]. In this paper, the focus was based on the principle where the signal is constructed with IMFs filtered using a Savitzky-Golayfilter [3], [4]. Our aim was to improve the method proposed in [3], [4], by filtering the noisy IMFs not by a single S-G filter, but by several cascaded S-G filters. Cascaded S-G filters are equivalent to passing the input signal through a S-G filter two or more times.

B. Savitzky-Golay Filter

Given N samples of a noisy IMF $c\eta(n)$ $\left(c\eta(0), c\eta(1), \cdots, c\eta(N-1)\right)$ and a window size of $2M+1$ points (frame size) centered at the central point of the interval, the smoothed value of $c\eta(i)$ by Savitzky-Golay filter is given by:

$$\hat{c}(i) = \sum_{j=-M}^{M} g(j) c\eta(i+j) \tag{4}$$

where $g(j)$ are filter coefficients.

$\hat{c}(i)$ is the average of the data points $c\eta(i\text{-}M), \cdots, c\eta(\text{-}1), c\eta(0), c\eta(1), \cdots, c\eta(i+M)$. The concept of Savitzky-Golay filtering is to find the coefficients $g(j)$ that preserve higher moments within the moving window by a higher order polynomial [6]. For a polynomial order and for each point $c\eta(i)$, the coefficients $g(j)$ must be determined optimally to minimize the least-squares error in fitting a polynomial to all $2M+1$ points in the moving window such that the $\hat{c}(i)$ *is the* corresponding value of the polynomial curve at position *i*.

IV. Results

The proposed denoising method was evaluated on four noisy simulated signals given in Fig. 1 (Doppler, Blocks, Bumps and Heavysine) corrupted by white Gaussian noise with *SNR* values of ± 5, ± 2 and 0 dB. Examples of noisy signals with an *SNR* of 2*dB* are shown in Fig. 2. The number of samples of the test signals was 2048. By fixing the window size to 41 and 2*dB SNR* before filtering of the Doppler signal, the right order of the SG filter was determined using 3 different orders (3, 4 and 6). Table I shows the obtained results for different orders using the original EMD-SG denoising and the EMD-SG multiple pass denoising (EMD Double SG: EMD-DSG, EMD Triple SG: EMD-TSG, EMD Quadruple SG: EMD-Q SG). One sees that the order 3 gives the best results on all of the denosing methods. Then, the parameters of the SG filter were 41 for size, and 3 for the order. The results of the proposed resulting from two, three and four passes were compared to original EMD-SG resulting from one pass. Table II-Table VI show the obtained results for different *SNR* before denoising. These results generally show that the output *SN*R after denoising increases with the number of passes.

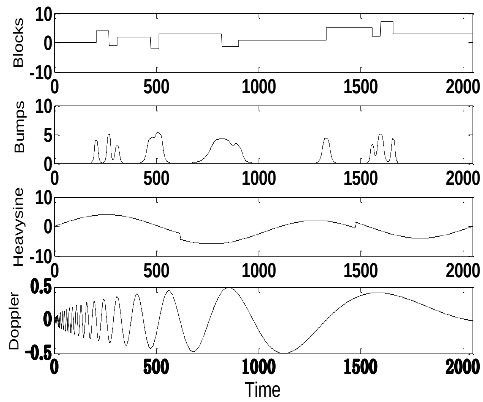

Figure 1. Test signals with T=2048.

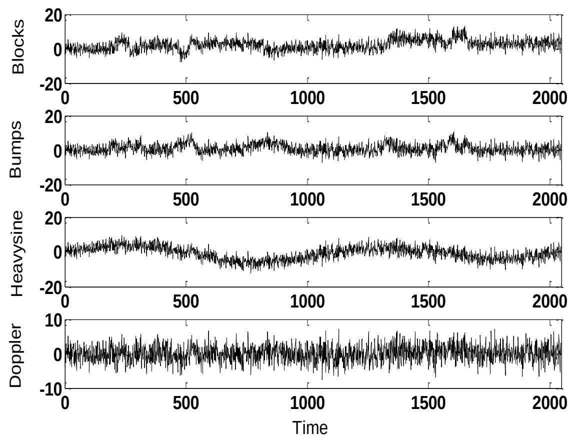

Figure 2. Noisy test signals *SNR* (2*dB*).

TABLE I. COMPARISONS OF THE ORDER OF SG FILTER (P) FOR DOPPLER SIGNAL WITH *SNR*=2DB

Methods	EMD_{SG}	EMD_{DSG}	EMD_{TSG}	EMD_{QSG}
Order of SG filter (p)	SNR[dB]	SNR[dB]	SNR[dB]	SNR[dB]
	MSE	MSE	MSE	MSE
(3)	12.7998	13.3074	13.2693	13.2380
	0.0045	0.0040	0.0040	0.0040
(4)	11.6666	12.3213	12.5388	12.6737
	0.0058	0.0050	0.0047	0.0046
(6)	10.6203	11.1004	11.3069	11.4489
	0.0074	0.0066	0.0063	0.0061

(before denoising)

TABLE II. COMPARISONS OF *MSE* AND *SNR* (AFTER DENOISING) VALUES FOR *SNR*=-5DB

Signal	Blocks	Bumps	Heavysine	Doppler
Methods	SNR[dB]	SNR[dB]	SNR[dB]	SNR[dB]
	MSE	MSE	MSE	MSE
EMD_{SG}	6.8169	6.9897	7.1995	6.9559
	1.8350	0.6480	1.8140	0.01729
EMD_{DSG}	7.6735	7.7430	8.1730	7.8260
	1.5065	0.5448	1.4497	0.0141
EMD_{TSG}	8.0257	7.9896	8.6167	8.1557
	1.3892	0.5147	1.3089	0.0131
EMD_{QSG}	8.2899	8.1290	8.9457	8.3889
	1.3072	0.4984	1.2134	0.0124

TABLE III. COMPARISONS OF *MSE* AND *SNR* (AFTER DENOISING) VALUES FOR *SNR*=-2DB

Signal	Blocks	Bumps	Heavysine	Doppler
Methods	SNR[dB]	SNR[dB]	SNR[dB]	SNR[dB]
	MSE	MSE	MSE	MSE
EMD_{SG}	9.4721	9.8309	10.1867	9.6511
	0.9957	0.3368	0.9118	0.0092
EMD_{DSG}	10.2336	10.4090	11.1583	10.4126
	0.8355	0.2948	0.7290	0.0078
EMD_{TSG}	10.5035	10.4986	11.5962	10.6247
	0.7852	0.2888	0.6591	0.0074
EMD_{QSG}	10.7104	10.4992	11.9222	10.7688
	0.7486	0.2888	0.6114	0.0071

TABLE IV. COMPARISONS OF *MSE* AND *SNR* (AFTER DENOISING) VALUES FOR *SNR*=0DB

Signal	Blocks	Bumps	Heavysine	Doppler
Methods	SNR[dB]	SNR[dB]	SNR[dB]	SNR[dB]
	MSE	MSE	MSE	MSE
EMD_{SG}	11.1022	11.6614	12.1711	11.3066
	0.6439	0.2210	0.5774	0.0063
EMD_{DSG}	11.7709	12.0590	13.1396	11.9573
	0.5626	0.2016	0.4620	0.0054
EMD_{TSG}	11.9630	11.9973	13.5710	12.0564
	0.5510	0.2045	0.4183	0.0053
EMD_{QSG}	12.1166	11.8726	13.8931	12.1191
	0.5422	0.2105	0.3884	0.0052

TABLE V. COMPARISONS OF *MSE* AND *SNR* (AFTER DENOISING) VALUES FOR *SNR*=2DB

Signal	Blocks	Bumps	Heavysine	Doppler
Methods	SNR[dB]	SNR[dB]	SNR[dB]	SNR[dB]
	MSE	MSE	MSE	MSE
EMD_{SG}	12.5760	13.4147	14.1456	12.7998
	0.4872	0.1475	0.3664	0.0045
EMD_{DSG}	13.1275	13.5667	15.1085	13.3074
	0.4291	0.1425	0.2935	0.0040
EMD_{TSG}	13.2250	13.3154	15.5299	13.2693
	0.4196	0.1510	0.2664	0.0040
EMD_{QSG}	13.3155	13.0438	15.8459	13.2380
	0.4109	0.1607	0.2477	0.0040

TABLE VI. COMPARISONS OF *MSE* AND *SNR* (AFTER DENOISING) VALUES FOR *SNR*=5DB

Signal	Blocks	Bumps	Heavysine	Doppler
Methods	SNR[dB]	SNR[dB]	SNR[dB]	SNR[dB]
	MSE	MSE	MSE	MSE
EMD_{SG}	14.4151	15.8332	17.0760	14.6475
	0.3190	0.0845	0.1866	0.0029
EMD_{DSG}	14.7597	15.4856	18.0217	14.8983
	0.2946	0.0916	0.1501	0.0027
EMD_{TSG}	14.6994	14.8988	18.4175	14.6360
	0.2988	0.1048	0.1370	0.0029
EMD_{QSG}	14.6888	14.3924	18.7166	14.4615
	0.2995	0.1178	0.1279	0.0030

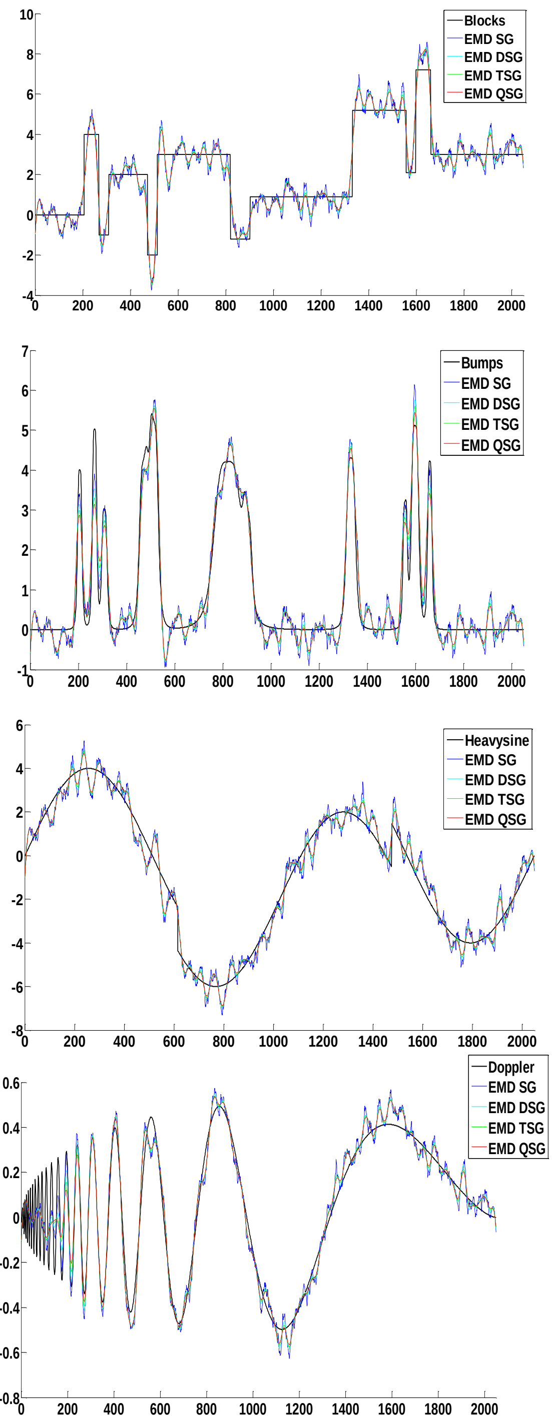

Figure 3. Denoising results in SNR (2dB) of test signals corrupted by Gaussian noise.

With less noisy signals of 2dB and 5dB, the obtained results by the three approaches depend on the characteristic of the signal. For example for the value of the *SNR* before 2*dB* (Table V and Fig. 3), the best result is obtained for heavysine signal with respect of *SNR* and the best result is obtained for Doppler signal with respect of *MSE*. For a given signal the *MSE* decreases with increasing *SNR*: $SNR_{\max} = 15.8459\ dB$ for $MSE \min = 0.2477$ regarding the heavysine signal. For different signals that rule is not verified. Indeed, $SNR_{\max} = 13.3074\ dB$ with $MSE \min = 0.0040$ for Doppler signal.

V. Conclusion

A signal denoising technique was presented based on Empirical Mode Decomposition and Multiple-pass Savitzky-Golay filter. The simulation results were in favour of the new EMD multiple pass denoising method in comparison with initial EMD-SG method. It is noteworthy that the multiple passes of the S-G filters were correspondingly slower, but still very quick.

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